A Generalized Approach to the Machining Economics Optimization

by

Mansoor Alam

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

MECHANICAL ENGINEERING

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Date Jun 13, 1988

DEDICATED TO MY MOTHER

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All Praise to Allah, the most merciful, the most beneficient.

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THESIS ABSTRACT

NAME : MANSOOR ALAM

TITLE OF STUDY : A GENERALIZED APPROACH TO THE MACHINING ECONOMICS OPTIMIZATION

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An extensive review of the machining optimization models has been carried out. Optimization methods pertinent to machining optimization have been compiled and evaluated by applying them to a number of selected machining models. Sequential Unconstrained Minimization Technique (SUMT) and a personal computer version of the Generalized Reduced Gradient (GRG2), known as the Generalized Integrated Optimizer (GINO), were found to be most suitable for application in machining economics optimization.

Taking multipass turning as a representative case of multipass machining, five cutting strategies have been proposed for multipass machining optimization. Cutting strategies are based on the optimal way of distributing the total depth of layer to be removed, over several passes required to do the job, while satisfying a given set of constraints. The performance of the proposed cutting strategies has been evaluated and their sensitivity has been demonstrated through a general machining optimization model to some of the process variables and constraints. A most economical machining strategy has been identified out of the proposed strategies. A general computer program package based on SUMT optimization method, which runs on both the mainframe and personal computer, has been developed to handle generalized multipass machining optimization problems. With some modifications, the developed program package is also capable of handling multi-tool and multi-operation optimization problems both in research and industrial environment.

MASTER OF SCIENCE DEGREE IN
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Dhahran, Saudi Arabia
January 1988

ملخص

الاســـم : منصور عالـــم

موضوع الدراســة : طريقة عامة الى ايجاد أمثل الحلول لاقتصاديات قطع المعادن

المجال الدراســي : هندسة ميكانيكية (تخصص انتاج)

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لقد تم حصر ومراجعة شاملة لنماذج ايجاد الظروف التشغيلية والاقتصادية المثلى لعمليات قطع المعادن . وقد تم حصر وتقويم طرق ايجاد أمثل الحلول ذات الصلة بعمليات القطع وتم تطبيقها لمجموعة مختارة من نماذج اقتصاديات القطع لتقويم أدائها في ايجاد أمثل الظروف التشغيلية لعمليات القطع . وقد وجد أن أنسب الطرق لايجاد الحلول المثلى لعمليات القطع هي تقنية التخفيف التتابعي غير المقيد (SUMT) وطريقة التدرج المخفض العامة رقم ٢ (GRG2) والمعروفة عند تطبيقها على الحاسب الآلي بطريقة الحل الأمثل التكاملي العام (GINO) .

وعندما أتخذت الخراطة متعدد الخطوات كمثال لعمليات قطع عمق معين في عدة خطوات تم اقتراح خمسة مداخل أو استراتيجيات للقيام بقطع طبقة ذات عمق معين من المعدن في عدة خطوات وبطريقة مثلى . وتقوم الاستراتيجيات على طريقة توزيع العمق الاجمالي للطبقة عبر الخطوات المتعددة المطلوبة لازالتها مع الايفاء بالقيود التشغيلية والاقتصادية لعملية القطع . وقد تم تطبيق هذه الاستراتيجيات على نموذج عام لاقتصاديات القطع لايجاد أفضل الحلول لعوامل التشغيل لكل استراتيجية مستخدمين SUMT و GINO لايجاد الحلول المثلى ، وبذلك تم التعرف على أفضل استراتيجية اقتصادية من بين هذه الاستراتيجيات لازالة عمق معين من المعدن في عدة خطوات . وللقيام بايجاد أمثل الظروف التشغيلية الاقتصادية لعمليات قطع المعادن تم تطوير برنامج حاسب آلي عام على أساس SUMT يمكن تنفيذه على الحاسب الآلي الرئيسي أو الشخصي ويساعد هذا البرنامج بالاضافة الى امكانية ايجاد الحلول المثلى لعمليات قطع المعادن في خطوات متعددة ، على حل المشاكل الخاصة بعمليات القطع ذات العدد المتعددة أو ذات عمليات القطع المتعددة .

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1. INTRODUCTION

The optimization objective of a machining operation is to determine optimal or best set of operating conditions which satisfy an objective subject to a number of operational constraints. Optimization also finds its application in adaptive control machining, where the variables that control the process are modified to optimize an objective function as the process state changes over time [1]. The optimization objectives considered in the literature include minimum production cost, minimum production time, maximum metal removal rate or maximum profit [2-6]. Both deterministic and probabilistic models of the objective functions and constraints have been dealt with by researchers in the field [7-9]. The machining processes that have been the subject of optimization include single-pass, multi-pass, multi-operations and multi-tool machining.

A host of optimization methods have been used to solve the machining economics problem. These include, but are not limited to geometric programming [10], dynamic programming [11] and sequential unconstrained minimization technique (SUMT) [12].

An extensive review of the machining optimization literature shows that researchers usually report the results of a specific optimization method applied to a given objective function and set of constraints of a particular machining operation. The investigators also address a particular machining situation which is usually selected to demonstrate the application of an optimization method to the machining economics problem. Another fact is that the researchers have not conducted extensive investigation to identify a single approach to handle multipass machining optimization. Since all machining operations involve the removal of a layer of material in more than one pass, the optimization of multipass machining operations is also constrained by the strategy which dictates the distribution of the layer of material to be removed, amongst the passes required to accomplish the operation.

Thus the objectives of this thesis are:

1. Survey and compilation of the machining optimization models (i.e. objective functions and constraint formulations) reported in the literature in order to characterise and compare the model objective functions and constraints.

2. Survey of the available optimization methods pertinent to the machining optimization problem. These methods are to be implemented on the machining optimization models compiled in objective 1 above to empirically evaluate their relative performance in handling machining optimization problems. The

most suitable methods are to be suggested for use in the machining optimization problems.

3. Optimization of multipass machining operations using a number of possible machining strategies of a multipass machining process. The proposed machining strategies are to be assessed for relative performance and sensitivity to selected process variables and constraints. A most general and economical approach to multipass machining optimization is to be identified.

4. Development of a software package capable of solving a generalized machining optimization model, using both the mainframe and personal computer and providing options for selection of different objective functions, constraints and machining strategies.

Chapter 2 of this thesis reviews the literature in the field of machining economics optimization. The chapter reports the objective functions, constraint formulations and optimization methods employed by investigators in this field.

Chapter 3 presents a number of optimization methods and their application to the machining optimization models selected from literature. For each selected model, results given by the optimization methods are compared to assess the relative performance of the methods in solving machining optimization problems. From the study the most suitable optimization methods are selected for use in machining optimization problems.

Chapter 4 proposes five machining strategies for multipass turning optimization. Using a general machining optimization model, the sensitivity of the strategies to selected process variables and constraints is carried out. Finally a most economical and general machining strategy is identified.

Chapter 5 describes the developed machining optimization computer program for multipass machining operations for various types of machining strategies. Sample runs of the program are also presented in the chapter.

Chapter 6 outlines the conclusions arising out of the present work and recommendations for future extensions.

2. LITERATURE REVIEW

The selection of economically optimal machining conditions has long been recognized as a major concern in metal cutting. The Taylors' tool life equation $VT^n = C$ is considered as the first intuitive attempt at the economic use of the machine tool system. Later Gilbert [13], first approached machining economics in a rational way and concluded that if cutting is done at very high speed, the machine and labor costs will be low, but the tool cost will be high. The opposite is true if cutting is done at very low speeds. This implies that there is an optimum cutting speed for any machining operation where the total machining cost per part is a minimum. Brown [14] presented some general relations for the selection of speed, feed and depth of cut to achieve optimum economic conditions in machining. Consideration was given to single point tools removing material in both one and two passes and to operations involving more than one tool. Minimum production cost and maximum production rate were the two objectives. The mathematical relationships developed by Brown are complex and require tedious calculations to arrive at definite conclusions. Considering only single pass machining, Brewer and Rueda [15] presented a number of nomograms to determine economic machining conditions for high speed steel, ceramic and carbide tools and a number of work materials.

Okushima & Hitomi [2] considered maximum profit as the objective and determined the cutting speed for a single pass turning operation. The process was unconstrained and simple differentiation was employed. Wu and Ermer [3] also considered the maximum profit rate as the objective. Their study is based on a fundamental principle of economics that maximum profit occurs at the production rate where the marginal revenue equals the marginal cost. This restricts the application of the analysis only to the cases where the marginal principle is taken into account. Boothroyd and Rusek [16] considered the condition for maximum rate of profit in machining. They concluded that this criterion is a convenient means of compromising between the conditions for minimum cost and for minimum production time in a machining process. Maximum profit rate for a single pass turning operation has also been considered by Armarego and Russell [17]. Simple differentiation was used and restrictions employed were considered separately and finally a speed-feed curve for maximum profit rate and various restrictions was obtained. Since profit is a direct function of the units sold, the maximum profit criteria is influenced by the supply and demand and the production cost.

Field et al [4] determined cost per piece and production rates for turning, milling, drilling, reaming and tapping operations. Crookall [18], using maximum production rate and minimum cost as the objective functions for a single pass turning operation, presented a performance envelope concept, representing the permissible and desireable operating regions for a particular combination of workpiece and tool. It demonstrates the multivariable nature of machining in which constraints, interdependencies and the necessity to compromise are the essence of parameter selection. Crookall and Venkataramani [19], related the cycle time of the turning operation to feed, speed and depth of cut, considering the possibility of two passes. Initial feed was considered as the independent parameter. The associated depths of cut, cutting speeds, final speed and cycle time were plotted against a common base of initial feed. Only two constraints were considered: power availability and surface finish requirements. Ermer [10], seeking to minimize the production cost for a single pass turning operation used geometric programming as the optimization technique. Constraints for feed, horsepower, and surface finish were considered and the degree of difficulty proceeds from zero to two. Petropolous [7] using geometric programming, minimized the unit cost of a single pass turning operation with only the power and surface roughness as the constraints. Walvekar and Lambert [20], have also discussed the use of geometric programming to determine the cutting speed and feed which yield minimum production cost. Ermer, Petropolous and Walvekar and Lambert [7,10,20] have infact tried to demonstrate geometric programming technique as a tool for machining optimization by applying it to simple machining economic problems. In another paper Lambert and Walvekar [21] have presented a two stage optimization procedure for a multipass machining operation ,where geometric programming was utilized to determine the values of the machining variable for each pass to yield minimum production cost. The method is demonstrated for a two pass operation only.

Ermer and Kromodihardjo [22] have illustrated that in turning, multipass operation may be more economical than single pass operation. Minimum cost was used as the optimizing objective and optimization was achieved by a combination of Geometric programming with Linear programming. Yellowley [23] has examined the economics of a two pass turning operation, addressing it in an analytical way. The influence of surface finish constraint and power constraint has been considered on optimal conditions. The paper by Yellowly [23] aims at finding guidelines for subdivisions of total depth of cut for a two pass turning operation. While relying on simple differentiation to obtain minimum production cost Filippi and Ippolito [24] have also considered the influence of power, maximum spindle speed and tool life constraints on optimization of cutting conditions for turning operations of cast iron with different tool materials.

All preceding investigators considered the machining optimization model (i.e. the objective function and the constraints) as deterministic in nature. Iwata et al [8] proposed a probabilistic approach in the determination of the optimum cutting conditions. The volume of material machined per unit of tool wear and production cost per component for a single pass turning operation were considered as the probabilistic objective functions. The constraints were both deterministic and probabilistic. A similar methodology for the determination of optimum machining conditions has been discussed by Hati and Rao [12]. Three objective functions i.e. cost of production per piece, production rate and total profit were considered separately. A deterministic model of the problem was formulated for a multipass turning operation first and then the probabilistic model was obtained. Results obtained from the deterministic approach were compared with those of the probabilistic approach for cost of production per piece and production rate. However the paper does not consider the final product quality i.e. the surface finish constraint. The optimization method employed in the two preceding References [8,12] was sequential unconstrained minimization (SUMT) technique.

Another paper by Iwata, Murotsu and Oba [9] deals with the optimization of cutting conditions for production cost of a multipass operation considering both deterministic and probabilistic nature of machining processes. An algorithm based on dynamic programming and stochastic programming was developed to handle the problem of optimizing the number of passes together with the cutting speed, feed and depth of cut at each pass for a given total depth of cut to be removed from a workpiece.

Unklesbay and Creighton [11] have used for the problem of optimizing multipass machining processes, geometric and dynamic programming techniques. The machining process was formulated as a dynamic programming problem in which the stages were considered to be the cutting process. Each pass was then optimized using geometric programming technique. Only one objective function, i.e. the machining cost per piece was considered. Davis et al [5] highlighted the optimization of machining parameters in a framework for adaptive computer control. A simplified dynamic programming rationale was developed to minimize the production time. The objective function and the constraints considered were deterministic in nature. Over-simplified assumptions, like considering tool life as a constant were taken into account.

A summarized account of the literature review presented in the foregoing paragraphs is shown in Tables 2-1 and 2-2. Table 2-1 lists author's name, year of publication, machining operation considered, objectives, constraints and optimization method employed. Table 2-2 mentions the chief characteristic of each paper. The references cited above, which handle constrained optimization, have used empirical relationships for objective function and constraint formulations for specific combination of workpiece and tool material. Most of the papers do not report the source of empirical relationships. Their interest is usually focused on demonstrating a machining optimization method by a numerical example.

A large number of empirical relationships for machining operations, were compiled by [25] from experimental data at different research laboratories. Expressions for cutting force ,tool life, and power were determined. Property of workpiece material like Brinell hardness and property of the tool material and geometry like lip angle was also included in some of the relationships. The values of exponents used in these relationships have been tabulated for a number of tool and workpiece material combinations. Kaczmarek [26] has also listed a number of empirical relationships for machining operations from experimental data. Inspite of the availability of these resourceful and factual relationships, most of the model and

TABLE 2-1 Summary of Literature Review

No.	Author	Year of Publication	Type of Operation	Objective	Constraints	Optimization Method
1.	Brown, R.H. [14]	1961	Turning Two pass	Minimum Production cost Maximum Production rate	Speed, feed, power	*
2.	Brewer, R.C. Reuda, R. [15]	1963	Turning Single pass	Minimum Production cost	Speed, feed, power	*
3.	Okushima, K. Hitomi, K. [2]	1964	Turning Single pass	Maximum profit	None	*
4.	Wu, S.M. Ermer, D.S [3]	1966	Turning Single pass	Maximum profit	Speed, feed	*
5.	Armarego, E.J Russell J.K. [17]	1966	Turning Single pass	Maximum profit	Speed, feed, cutting force power	*
6.	Field et al [4]	1968	Turning, Drilling, milling, reaming, tapping	Minimum Production cost Maximum Production rate	None	*

*Differentiation alone or combined with graphical presentation is employed.

TABLE 2-1 (continued)

No.	Author	Year of Publication	Type of Operation	Objective	Constraints	Optimization Method
7.	Crookall, J.R [18]	1969	Turning Single pass	Minimum Production cost Maximum Production rate	Speed, feed, power, surface finish, work piece rigidity	*
8.	Walvekar, A.G. Lambert, B.K. [20]	1970	Turning Single pass	Minimum Production cost	Speed, feed, power, surface finish	Geometric programming
9.	Ermer, D.S. [10]	1971	Turning Single pass	Minimum Production cost	feed, power, surface finish	Geometric programming
10.	Crookall, R. Venkataramani [19]	1971	Turning Multipass	Maximum Production rate	Power, surface finish, work piece deflection	*
11.	Iwata et al [8]	1972	Turning Singlepass	Minimum Production cost Maximum volume of material removed per unit of tool wear	Speed, feed, force, power surface finish, stable cutting region	Sequential unconstrained minimization technique

*Differentiation alone or combined with graphical presentation is employed.

TABLE 2-1 (continued)

No.	Author	Year of Publication	Type of Operation	Objective	Constraints	Optimization Method
12.	Petros G. Petropolous [7]	1972	Turning singlepass	Minimum Production cost	Power, surface finish	Geometric programming
13.	Hati, S.K. Rao, S.S. [12]	1975	Turning Multipass	Minimum Production cost Maximum Production rate Maximum Profit	Speed, feed, depth of cut, force, power, tool temperature	Sequential unconstrained minimization technique
14.	De Filippi Ippolito, R. [24]	1975	Turning Singlepass	Minimum Production cost	Speed, power, tool life, surface finish	*
15.	Boothroyd, G. Rusek, P. [16]	1976	General Machining	Maximum Profit	None	*
16.	Iwata, K. Murotsu, Y. Oba, F. [9]	1977	Turning Multipass	Minimum Production cost	Speed, feed, cutting force, power surface finish, stability, depth of cut	Optimization involving dynamic and stochastic programming

*Differentiation alone or combined with graphical presentation is employed.

TABLE 2-1 (continued)

No.	Author	Year of Publication	Type of Operation	Objective	Constraints	Optimization Method
17.	Walvekar, A. Lambert, B.K [21]	1978	Turning Multipass	Minimum Production cost	Speed, feed, power, surface finish	Geometric programming
18.	Unklesbay, K. Creighton, D. [11]	1978	Turning Multipass	Minimum Production cost	Speed, feed, force, power, surface finish	Dynamic and geometric programming combined
19.	Davis et al [5]	1980	Turning Multipass	Maximum Production rate	Speed, feed, depth of cut, tool thrust, Power, torque	Staged optimization rationale based on dynamic programming
20.	Ermer, D.S. Kromodihardjo [22]	1981	Turning Multipass	Minimum Production cost	Surface finish, power, feed	Geometric and linear programming implemented through a computer program
21.	Yellowly, I. [23]	1982	Turning Multipass	Minimum Production cost	Surface finish, power	*

*Differentiation alone or combined with graphical presentation is employed.

TABLE 2-2 Summary of Literature Review

No.	Author	Year of Publication	Chief Characteristic
1.	Brown, R.H. [14]	1961	Differentiation is highly involved and requires tedious calculations
2.	Brewer, R.C. Rueda, R. [15]	1963	Nomograms have been presented for HSS, ceramic and carbide tools and a number of work materials
3.	Okushima, K. Hitomi, K. [2]	1964	An expression is deduced for maximum profit cutting speed. The results were not applied to practical situation and no conclusions were drawn.
4.	Wu, S.M. Ermer, D.S. [3]	1966	The study is applicable to the cases where the marginal revenue equals marginal cost.
5.	Armarego, E.J Russell, J.K. [17]	1966	Restrictions on machining conditions are employed separately and finally a speed-feed curve for maximum profit and various restrictions is obtained.
6.	Field et al [4]	1968	Generalized equations for cost and production rate are presented for turning, milling, drilling, reaming and tapping. All these equations were programmed in a computer for calculation of the objectives for specific parts, operations and machine tool combinations.
7.	Crookall, J.R [18]	1969	A performance envelope is developed for a particular combination of workpiece and tool. The effects of various constraints in limiting the operating range is examined.

TABLE 2-2 (continued)

No.	Author	Year of Publication	Chief Characteristic
8.	Walvekar, A.G. Lambert, B.K. [20]	1970	The paper demonstrates the application of geometric programming to machining optimization.
9.	Ermer, D.S. [10]	1971	This paper also demonstrates the application of geometric programming to machining optimization.
10.	Crookall, R. Venkataramani [19]	1971	The variation in cutting conditions i.e. speed, feed and depth of cut is described by graphical presentations. The results are applicable to particular set of workpiece and tool combinations.
11.	Petros G. Petropolous [7]	1972	This paper highlights the application of geometric programming to the field of machining optimization.
12.	Iwata et al [8]	1972	The objective functions and the constraints were considered probabilistic.
13.	Hati, S.K. Rao, S. [12]	1975	Integral number of passes were considered thereby assigning depth of cut for each pass. The results were compared for deterministic and probabilistic approaches.
14.	De Filippi Ippolito, R. [24]	1975	The influence of some constraints on the optimum cutting conditions was analysed for given combination of tool and workpiece material.

TABLE 2-2 (continued)

No.	Author	Year of Publication	Chief Characteristic
15.	Boothroyd,G. Rusek,P. [16]	1976	Maximum rate of profit is determined and analysis presented for the effect of worker incentive schemes and batch production on the machining conditions.
16.	Iwata,K. Murotsu Y. Oba,F.[9]	1977	Speed,feed,depth of cut and the number of passes are optimized simultaneously.
17.	Walvekar,A. Lambert B.K. [21]	1978	A two stage optimization procedure using geometric programming for optimization at each stage is presented
18.	Unklesbay,K. Creighton,D.L. [11]	1978	The application of dynamic staged optimization to a multipass turning operation is shown.
19.	Davis et al [5]	1980	The approach of simplified dynamic rationale is developed for application to adaptive computer control.
20.	Ermer,D.S. Kromodihardjo [22]	1981	It has been shown that under assumed values of constraints,multipass operation can be cheaper than the single pass.
21.	Yellowly,I. [23]	1982	The paper aims at finding guidelines for subdivisions of total depth of cut for a two pass turning operation.

constraint formulations used by researchers were not drawn from these relationships. This is due to the fact that the convention is to demonstrate the application of selected optimization models to an objective function by a relevant example. A selected number of constraint formulations from [25,26,39,41,42] are presented in the Appendix A.

The preceding literature review points to the fact that researchers do not perform comparisons of their work in terms of models and optimization methods. The present work tests the available optimization methods by applying them to selected models from literature and comparing the results.

The literature review also suggests that the investigators in the field of machining economics have been limited in their approach, each one addressing a specific machining situation. The work of this thesis considers a number of machining situations offering several operational and economic options to choose from. The most economical approach is also suggested.

3. OPTIMIZATION METHODS FOR MACHINING OPERATIONS

This chapter presents a review of the available optimization methods which have been applied to machining optimization or have the potential of application. The major features of the methods are outlined and their relative performances in solving a number of machining models from literature are evaluated. Section 3.1 contains an overview of the optimization methods applicable to machining operations whereas section 3.2 presents the solutions of selected machining optimization models by the optimization methods.

3.1 REVIEW OF THE OPTIMIZATION METHODS

In general, optimization methods are classified as single variable unconstrained, single variable constrained, multivariable unconstrained and multivariable constrained methods. The choice of a method depends on the model i.e. the objective function and the constraints, which can be linear or non-linear or combination of the two. The constraints can also be of the equality or inequality type.

In metal cutting operations, the objective functions i.e. the production cost, the production time, profit and metal removal rate are non-linear functions of the machining conditions such as speed, feed, depth of cut or tool geometry parameters. Since the non-linear objective functions are bounded by a combination of equality and inequality constraints which could be linear or non-linear, multivariable constrained optimization based on non-linear programming (NLP) techniques are well suited to machining optimization models. A large number of non-linear programming techniques are available in the literature [27,28,29,30]. In this thesis, the optimization methods and algorithms which are considered for solving selected machining optimization models from the literature are: the Sequential Unconstrained Minimization Technique (SUMT) [31,34], the Complex Algorithm [31,32], the Hill Algorithm [33], the Generalized Reduced Gradient (GRG2) Algorithm [36], and the Gomtry Algorithm [31,37].

3.1.1: Sequential Unconstrained Minimization Technique

This program finds the minimum of a multivariable, nonlinear function subject to non-linear inequality and equality constraints. The objective function and constraints are written as:

Minimize $F(X_1 , X_2 ,\ldots, X_N)$ (3.1)

Subject to $G_k(X_1 , X_2 ,\ldots, X_N) \geq 0$ (3.2) for $k = 1,2,\ldots M$.

and $$H_k(X_1,X_2,\ldots,X_N) = 0 \tag{3.3}$$

for $k = M+1, M+2, \ldots, M+P$

where $X_1,X_2,\ldots,X_N$ are optimization variables, M = Number of inequality constraints and P = Number of equality constraints.

The technique used in this algorithm was developed by Fiacco and McCormick [35]. It transforms the general constrained non-linear problem to multivariate unconstrained problem, which is solved by one of the methods used for solving the unconstrained optimization problem. The options available are the generalized Newton Raphson method, the steepest descent method and a modification of the Fletcher - Powell method. The computer program was developed by W. Charles Mylander, R.L. Holmes and G.P. McCormick [31]. This program makes use of first and second derivatives, but the choice for determining and checking derivatives depends on the user. It accepts the starting vector i.e. initial values of the model variables as an input to the program, which may be feasible or infeasible.

A modified objective function is formulated consisting of the original function and penalty functions with the form:

$$P = F - r\sum_{k=1}^{M} \ln G_k + \sum_{k=M+1}^{M+P} \frac{H_k^2}{r} \tag{3.4}$$

where 'r' is a positive constant. As the algorithm progresses, r is revaluated to form a monotonically decreasing sequence $r_1 > r_2 > \cdots > 0$, P is minimized, new value of 'r' is selected and the procedure repeated until the convergence criteria is satisfied.

Iwata et al [9], and Hati and Rao [12] have used SUMT analytically for machining optimization problem.

3.1.2: Complex or Box Algorithm

This program finds the maximum of a multivariable, nonlinear function, subject to non-linear inequality constraints.

Maximize $F(X_1, X_2, .., X_N)$ (3.5)

Subject to $G_k \leq X_k \leq H_k$ (3.6)

where $k = 1, 2, .., M$

The implicit variables $X_{N+1}, \cdots, X_M$ are dependent functions of the explicit independent variables $X_1, X_2, \cdots, X_N$. The upper and lower constraints H_k and G_k are either constants or functions of the independent variables. The program was developed by Joel A. Richardson, Arizona State University [31].

This algorithm is a sequential search technique based on the complex method of BOX [32]. The procedure tends to find the global maximum (minimum) due to the fact that the initial set of points are randomly scattered throughout the feasible region. An original complex of $K \geq N+1$ points is generated consisting of a feasible starting point and k-1 additional points, generated from random numbers and constraints for each of the independent variables given as:

$$X_{i,j} = G_i + r_{i,j} (H_i - G_i) \tag{3.7}$$

where $i= 1,2,\ldots,N$

and $j= 1,2,\ldots,K-1$

where $r_{i,j}$ are random numbers between 0 and 1.

The selected points must satisfy both the explicit and implicit constraints. In case, constraints are violated, correction factors are applied. The objective function is evaluated at each point and convergence is assumed when the objective function values at each point are within b units (small number, magnitude of function times 10^{-4}) for a number of interations. No derivatives are required. The algorithm accepts only feasible starting points. Abuelnaga [30] has reported using the method of BOX for optimization of stepped part turning.

3.1.3: Hill Algorithm

This program finds the maximum or minimum of a multivariable, nonlinear function, subject to non-linear inequality constraints.

Maximize or Minimize $\quad F(X_1, X_2, \cdots, X_N) \qquad (3.8)$

Subject to $\quad G_k \le X_k \le H_k \qquad (3.9)$

where k = 1,2,..,M

The implicit variables $X_{N+1}, \cdots, X_M$ are dependent functions of the explicit independent variables $X_1, X_2, \cdots, X_N$. The upper and lower constraints H_k and G_k are either constants or functions of the independent variables.

The computer program was developed by C.B. Yancey and R.C.

Spear [31] and the procedure is based on the "automatic" method proposed by Rosenbrock [33]. The method is a sequential search technique, and no derivatives are required . The objective function is evaluated and modified for constraint violation and boundry-zone violation. The boundry zones are defined as

$$\text{Lower Zone:} \quad G_k \leq X_k \leq G_k + (H_k - G_k) \times 10^{-4} \tag{3.10}$$

$$\text{Upper Zone:} \quad H_k \geq X_k \geq H_k - (H_k - G_k) \times 10^{-4} \tag{3.11}$$

If an improvement in the objective function is obtained without violating the boundary zones or constraints, the procedure is continued and the search is terminated when the convergence criteria is satisifed. The algorithm accepts feasible starting points only.

3.1.4: Gomtry Algorithm

This program finds the minimum of a multivariable, nonlinear function of geometric form

$$\text{Minimize :} \quad y_o(x) = \sum_{t=1}^{T_o} \sigma_{ot}\, C_{ot} \prod_{n=1}^{N} x_n^{a_{otn}} \tag{3.12}$$

Subject to constraints of the geometric form

$$\sum_{t=1}^{T_m} \sigma_{mt} C_{mt} \prod_{n=1}^{N} X_n^{a_{mtn}} \leq \sigma_m \tag{3.13}$$

for m = 1,2,...,M.

σ_{ot} and $\sigma_{mt} = \pm 1$ is the sign of each term in the objective function and mth constraint respectively.

C_{ot} and C_{mt} are the coefficients of each term in the objective function and mth constraint respectively.

$x_n > 0$ are the independent variables

$\sigma_m = \pm 1$ the constant bound of the mth contraint.

a_{otn} and a_{mtn} are the exponents of the nth independent variables of the t_{th} term of the objective function and m_{th} constraints.

M is the number of constraints

T_o is the number of terms in the objective function.

$T_1, T_2, \cdots, T_M$ are the number of terms in each constraint 1 to M respectively

$\sigma = \pm 1$, assumed sign of the objective function.

The algorithm originates from the geometric programming technique of Zener [37] and programmed by Garcia and Hogg [31]. Geometric programming finds the optimal way to distribute the total cost among the various terms of the objective function first, instead of seeking the optimum values of the optimization variables. After the optimal allocations are found, the optimal cost can be obtained and then the values of the optimization variables for the optimal cost are determined. For machining optimization Ermer [10], Petropolous [7], Walvekar and Lambert [20,21], and Unklesbay and Creighton [11] have used this technique analytically while Ermer and Kromodihardjo [22] have implemented geometric programming with linear programming through a computer program.

3.1.5: Generalized Reduced Gradient (GRG2) Alogrithm

A personal computer version of the generalized reduced gradient (GRG2) known as the generalized integrated optimizer (GINO) is used to solve the machining optimization models. GRG2 uses search techniques like the BFGS (Broyden, Fletcher, Goldfarb and Shanno) quasi-Newton method and the conjugate gradient methods. User can choose the search method.

GINO provides the user with considerable control over the tolerances used in determining whether various processes have converged. In addition to this, the user can control the number of interations. GINO also accepts initial guess values, if the user desires so. GINO output indicates whether the solution is feasible or not. When the inequalities of the model have not been strictly satisifed, it indicates slack or surplus. It also predicts the improvement in the objective function value if there is small but feasible change in the corresponding variable.

Since the review of the machining optimization literature has indicated that generalized reduced gradient (GRG2) has not been previously applied to machining optimization problems, its PC version (GINO) is included in the present work. This is supported by the fact that GRG2 is generally more effective than the penalty function algorithms and significantly faster and able to obtain a high level of accuracy in final solution [28].

3.1.6: Preparation of the Models for the Optimization Methods

The machining optimization models are input to GINO as they are written. For SUMT, a separate subroutine is needed to enter the model. While the objective function is translated directly into Fortran, the constraints are entered with their upper or lower bounds or both. For upper bounds, the limiting value is subtracted from the constraint expression. For lower bounds the constraint expression is subtracted from the limiting value. HILL algorithm needs four small subroutines for a model to be entered. One subroutine is for the objective function, another for the constraint formulations and one each for upper and lower bound values. The COMPLEX or BOX algorithm has one subroutine for the objective function and another for the constraint expressions which are entered along with the limiting values. GOMTRY algorithm does not need any subroutine for the model input. Instead the constraint expressions are expressed in a manner so that the right hand side is a unity. Then the coefficients and the powers of each term in the objective function and the constraints are entered as data input along with the number of variables. Complete details can be gathered from [31].

3.1.7: Comparative Features of the Optimization Methods

Sequential unconstrained minimization technique (SUMT) makes use of first and second derivatives. Algorithms of HILL and BOX are search methods which do not require derivatives. Generalised integrated optimizer (GINO) implements a version of the generalised reduced gradient algorithm (GRG2). While algorithms of BOX and HILL require a feasible starting vector, SUMT accepts both feasible or infeasible starting vector. GINO does not need a starting vector but accepts it when supplied. Geometric programming algorithm (GOMTRY) is different from the other optimization methods because it first finds the optimum allocations for each term of the objective function and then determines the values of the optimization variables. To input the model i.e. objective function and constraints into the optimization methods, SUMT, BOX and HILL need one, two and four subroutines respectively. GINO needs simply the model as the input and is the easiest to use in this respect.

Table 3-1 lists the characteristics of the optimization methods considered in this section. The Table indicates the type of constraints and the type of objective function which can be handled by a given optimization method. Table 3-2 shows the merits of the optimization methods in terms of the requirement of the input vector, the feasibility or infeasibility of the input vector, the number of user supplied subroutines and approximate number of cards in each method.

TABLE 3-1 Characteristics of Optimization Methods [30]

Optimization Method / Characteristic	SUMT	BOX	HILL	GOMTRY	GINO
Equality Constraints	+				+
Inequality Constraints	+	+	+	+	+
Linear Constraints	+				+
G:P: Constraints	+			+	+
Nonlinear Constraints	+	+	+		+
G.P. Obj. Function	+			+	+
Nonlinear Obj. Function	+	+	+		+
Feasibility Check	+	+	+		+

+: Characteristic is applicable to the optimization method
G.P.: Geometric programming

TABLE 3-2 Merits of Optimization Methods

Optimization Method	Input Vector Needed?	Input Vector Feasible or Infeasible?	User Supplied Subroutines	Approx: No. Of Cards
SUMT	Yes	Both	One	1553
BOX	Yes	Feasible	Two	250
HILL	Yes	Feasible	Four	258
GOMTRY	Yes	Both	None	361
GINO	Can be Supplied	Both	None Used	PC Code

3.2 EVALUATION OF THE OPTIMIZATION METHODS USING SELECTED MACHINING OPTIMIZATION MODELS

In this section machining optimization models selected from literature are solved with the optimization methods listed in section 3.1 and the results are used to compare the performance of the optimization methods.

3.2.1: Iwata, Murotsu and Oba Model [9]

For multipass turning operation of medium carbon steel using a carbide tool, with production cost (yens/piece) as the objective, Iwata, Murotsu and Oba [9] have presented the following unit cost model for minimization.

$$\text{Cost}=\sum_{i=1}^{n} 3927\times V_i^{-1}\times f_i^{-1}+1.95E\text{-}8\times V_i^{2.887}\times f_i^{-1}\times e^{5.884 f_i}\times d_i^{-1.117}+60 \tag{3.14}$$

Subject to the following constraints:

i) The maximum and minimum cutting speeds

$$14.13 \le V \le 1005.3 \quad \text{m/min} \tag{3.15}$$

ii) The maximum and minimum feeds

$$0.01 \le f \le 5.6 \quad \text{mm/rev} \tag{3.16}$$

iii) The maximum and minimum depth of cut

$$0 < d \le A \text{ mm} \tag{3.17}$$

where 'A' is the total depth of material to be removed.

iv) The maximum cutting force

$$F_c \le 170 \text{ kg} \tag{3.18}$$

where, $$F_c = 290.73 \times V^{-0.1013} \times f^{0.725} \times d \tag{3.19}$$

v) The maximum power consumption

$$P_c \le 7.5 \text{ kW} \tag{3.20}$$

where, $$P_c = \frac{F_c \times V}{4896} \tag{3.21}$$

vi) The stable cutting region related to the cutting surface

$$f\,V^2 \ge 2230.5 \tag{3.22}$$

vii) The maximum allowed surface roughness

$$0.356 \times f^2 \leq H_{max} \tag{3.23}$$

where H_{max} ranges from 0.01 to 0.06 mm.

viii) The sum of depths of cut of the 'n' passes used to remove the total depth 'A' of the material. i.e.

$$A = \sum_{i=1}^{n} d_i \tag{3.24}$$

The model of Equations (3.14) to (3.24) was solved using the optimization methods listed in section 3.1 for the minimum production cost, speed and feed for removing a layer of A=2 mm in one pass and subject to a surface finish constraint of H_{max}=0.06 mm. The results are shown in Table 3-3 for four input vectors arbitrarily selected from a 2^2 (2 level-2 variable) design matrix.

It can be seen from Table 3-3 that SUMT, BOX and GINO gave identical values of 108.03 yens/piece, the minimum production cost for the four input vectors. Although BOX algorithm needs a feasible input vector unlike GINO and SUMT, it provides optimal values of the variables similar to GINO and SUMT. HILL's results vary slightly with the input feasible vector for this model. For example for an

TABLE 3-3 Results of Solving Iwata's Model with Different Optimization Methods at Four Input Vectors

Optimization Method	Input Vector speed V m/min	Input Vector feed f mm/rev	Output speed V m/min	Output feed f mm/rev	Output Production cost yens/piece	Depth of cut d mm	Number of passes n
SUMT	190.0	0.23	216.08	0.388	108.03	2	1
	200.0	0.23	216.08	0.388	108.03	2	1
	190.0	0.32	216.08	0.388	108.03	2	1
	200.0	0.32	216.08	0.388	108.03	2	1
BOX	190.0	0.23	216.08	0.388	108.03	2	1
	200.0	0.23	216.08	0.388	108.03	2	1
	190.0	0.32	216.08	0.388	108.03	2	1
	200.0	0.32	216.08	0.388	108.03	2	1
HILL	190.0	0.23	203.95	0.385	110.99	2	1
	200.0	0.23	216.03	0.388	108.02	2	1
	190.0	0.32	206.73	0.386	110.27	2	1
	200.0	0.32	216.11	0.388	108.02	2	1
GINO	190.0	0.23	216.08	0.388	108.03	2	1
	200.0	0.23	216.08	0.388	108.03	2	1
	190.0	0.32	216.08	0.388	108.03	2	1
	200.0	0.32	216.08	0.388	108.03	2	1

input vector of 190 m/min and 0.23 mm/rev the cost given by HILL is 110.99 yens/piece. For another input vector of 200 m/min and 0.23 mm/rev HILL yields a production cost of 108.02 yens/piece which is the same as given by GINO, SUMT and BOX.

The results of Table 3-3 show that the minimum cost from the HILL's method for two input vectors differ by about 2.75% in comparison to the costs given by GINO, SUMT and BOX. Since no figures for standard error are available, and the results may vary with the change in any of the exponents in the model, it can be said that for the model by [9] the results given by all the optimization methods can be similar.

3.2.2: Hati and Rao Model [12]

This model used the following production cost in dollars/piece for optimizing multipass turning of a mild steel workpiece using a carbide tool.

$$\text{Cost} = n\,(3141.59 \times V^{-1} \times f^{-1} + 2.879E\text{-}8 \times V^{4} \times f^{0.75} \times d^{0.75} + 10) \qquad (3.25)$$

Subject to the following constraints:

i) The maximum and minimum cutting speed

$$50 \le V \le 400 \quad \text{m/min} \qquad (3.26)$$

ii) The maximum and minimum feed

$$0.30 \le f \le 0.75 \quad mm/rev \tag{3.27}$$

iii) Allowable depths of cut range

$$1.20 \le d \le 2.75 \ mm \tag{3.28}$$

iv) Cutting Force

$$F_c \le 85 \ kg \tag{3.29}$$

where,

$$F_c = (28.10V^{0.07} - 0.525V^{0.5})df[1.59 + 0.946\frac{1+x}{\sqrt{(1-x)^2+x}}] \tag{3.30}$$

and $x = [\frac{V}{142} \times e^{2.21f}]^2$ (3.31)

v) Cutting Power

$$P_c \le 2.25 \ kW \tag{3.32}$$

where, $$P_c = \frac{0.746F_cV}{4500} \tag{3.33}$$

vi) Tool life

$$25 \le TL \le 45 \text{ mins} \tag{3.34}$$

where, $$TL = 60 \times \frac{10^{10}}{V^5 \times f^{1.75} \times d^{0.75}} \tag{3.35}$$

vii) Temperature

$$T \le 1000 \ {}^\circ C \tag{3.36}$$

where, $$T = 132 \times V^{0.4} \times f^{0.2} \times d^{0.105} \tag{3.37}$$

viii) Limitation on the value of the depth of cut

$$\frac{A}{d} = n \tag{3.38}$$

where 'A' is the total depth to be removed and 'n' is the number of passes.

Table 3-4 shows the results of the minimum production cost, and optimal speed and feed from SUMT, BOX, HILL and GINO for this model for removing a total depth of cut of 5 mm in two passes with four different input vectors. SUMT gives the same value of 150.60 dollars/piece cost for the four input vectors. GINO also gives equal costs of 151.55 dollars/piece for the four input vectors. For the

TABLE 3-4 Results of Solving Hati and Rao's Model with Different Optimization Methods at Four Input Vectors.

Optimization Method	Input Vector speed V m/min	Input Vector feed f mm/rev	Output speed V m/min	Output feed f mm/rev	Output Production cost dollars/piece	Depth of cut d mm	Number of passes n
SUMT	146.0	0.375	154.13	0.379	150.60	2.5	2
	196.0	0.375	154.18	0.379	150.60	2.5	2
	146.0	0.575	154.18	0.379	150.60	2.5	2
	196.0	0.575	154.18	0.379	150.60	2.5	2
BOX	146.0	0.375	146.15	0.375	151.60	2.5	2
	196.0	0.375	Input vector infeasible, no solution found				
	146.0	0.575	Input vector infeasible, no solution found				
	196.0	0.575	Input vector infeasible, no solution found				
HILL	146.0	0.375	146.36	0.375	151.64	2.5	2
	196.0	0.375	Input vector infeasible, no solution found				
	146.0	0.575	Input vector infeasible, no solution found				
	196.0	0.575	Input vector infeasible, no solution found				
GINO	146.0	0.375	151.55	0.375	151.55	2.5	2
	196.0	0.375	151.55	0.375	151.55	2.5	2
	146.0	0.575	151.55	0.375	151.55	2.5	2
	196.0	0.575	151.55	0.375	151.55	2.5	2

first input vector of 146 m/min and 0.375 mm/rev BOX and HILL give production costs of 151.60 and 151.64 dollars/piece respectively which are almost equal to the cost of 151.55 given by GINO. For the other three input vectors BOX and HILL indicate their infeasibility and no solutions were found.

3.2.3: Petropolous Model [7]

The production cost (pence/piece) for single pass turning of medium carbon steel workpiece using a carbide tool is given by:

$$\text{Cost} = 452 \times V^{-1} \times f^{-1} + 1E\text{-}5 \times V^{2.33} \times f^{0.4} \qquad (3.39)$$

It is required to obtain the minimum cost subject to the following constraints:

i) Cutting Power:

$$P_c \leq 5.5 \qquad (3.40)$$

where, $$P_c = 10.6E\text{-}2\ V \times f^{0.83}\ \text{kW} \qquad (3.41)$$

ii) Surface Finish:

$$R_a \leq 2\ \mu\text{meters} \qquad (3.42)$$

where, $$R_a = 2.2E4 \times V^{-1.52} f \quad \mu\text{meters} \tag{3.43}$$

The model by Petropolous [7] for machining a 3 mm layer in one pass was solved by SUMT, GINO, BOX and HILL optimization methods and the results are listed in Table 3-5 at four different input vectors. Only SUMT and GINO were found to be insensitive to the input vectors whether they are feasible or infeasible. For the first input vector of 185 m/min and 0.15 mm/rev, BOX and HILL give costs which have an error of 14.45% and 5.2% respectively relative to the cost of 12.097 given by SUMT and GINO. BOX and HILL fail to find a solution for the fourth input vector due to its infeasibility.

3.2.4: Ermer Model [10]

The objective of production cost (dollars/piece) for single pass turning is given as:

$$\text{Cost} = 1.25 \times V^{-1} \times f^{-1} + 1.8E\text{-}8 \times V^{3} \times f^{0.16} + 0.2 \tag{3.44}$$

Constraints are given by:

i) Feed

$$f \le 0.01 \text{ ipr} \tag{3.45}$$

TABLE 3-5 Results of Solving Petropolous's Model with Different Optimization Methods at Four Input Vectors

Optimization Method	Input Vector		Output			Depth of cut d mm	Number of passes n
	speed V m/min	feed f mm/rev	speed V m/min	feed f mm/rev	Production cost pence/piece		
SUMT	185.0	0.15	174.38	0.232	12.097	3	1
	215.0	0.15	174.38	0.232	12.097	3	1
	185.0	0.20	174.38	0.232	12.097	3	1
	215.0	0.20	174.38	0.232	12.097	3	1
BOX	185.0	0.15	250.51	0.138	13.845	3	1
	215.0	0.15	250.53	0.150	13.845	3	1
	185.0	0.20	197.33	0.200	12.620	3	1
	215.0	0.20	Input vector infeasible, no solution found				
HILL	185.0	0.15	201.89	0.194	12.727	3	1
	215.0	0.15	219.09	0.176	13.120	3	1
	185.0	0.20	197.23	0.200	12.620	3	1
	215.0	0.20	Input vector infeasible, no solution found				
GINO	185.0	0.15	174.38	0.232	12.097	3	1
	215.0	0.15	174.38	0.232	12.097	3	1
	185.0	0.20	174.38	0.232	12.097	3	1
	215.0	0.20	174.38	0.232	12.097	3	1

ii) Surface finish

$$SF \le 100\ \mu\text{inches} \tag{3.46}$$

where, $$SF = 1.36E10\ V^{-1.52} f^{1.004} \tag{3.47}$$

iii) Horse power

$$Hp \le 2.0 \tag{3.48}$$

where, $$Hp = 3.58\ V^{0.91} f^{0.78} \tag{3.49}$$

This model was solved by GINO, SUMT, BOX and HILL optimization methods for a total depth of cut of 0.2 inches in one pass using four different input vectors and the results are listed in Table 3-6. SUMT and GINO come up with the same output of 6.255 dollars/piece with any feasible or infeasible input vector. BOX and HILL have been found to be sensitive to the input vector when compared with GINO and SUMT which are insensitive. Two of the input vectors have been found to be infeasible for both HILL and BOX methods.

3.2.5: Ermer and Kromodihardjo Model [22]

The objective of production cost (dollars/piece) for single pass

TABLE 3-6 Results of Solving Ermer's Model with Different Optimization Methods at Four Input Vectors

Optimization Method	Input Vector speed V fpm	Input Vector feed f ipr	Output speed V fpm	Output feed f ipr	Output Production cost dollars/piece	Depth of cut d in	Number of passes n
SUMT	135.0	0.0011	143.90	0.0014	6.255	0.2	1
	170.0	0.0011	143.90	0.0014	6.255	0.2	1
	135.0	0.0035	143.90	0.0014	6.255	0.2	1
	170.0	0.0035	143.90	0.0014	6.255	0.2	1
BOX	135.0	0.0011	181.13	0.0011	6.508	0.2	1
	170.0	0.0011	181.11	0.0011	6.508	0.2	1
	135.0	0.0035	Input vector infeasible, no solution found				
	170.0	0.0035	Input vector infeasible, no solution found				
HILL	135.0	0.0011	151.80	0.0013	6.312	0.2	1
	170.0	0.0011	181.11	0.0011	6.507	0.2	1
	135.0	0.0035	Input vector infeasible, no solution found				
	170.0	0.0035	Input vector infeasible, no solution found				
GINO	135.0	0.0011	143.90	0.0014	6.255	0.2	1
	170.0	0.0011	143.90	0.0014	6.255	0.2	1
	135.0	0.0035	143.90	0.0014	6.255	0.2	1
	170.0	0.0035	143.90	0.0014	6.255	0.2	1

turning is given by:

$$\text{Cost} = 1.2566 \times V^{-1} \times f^{-1} + 1.687\text{E-}7 \times V^{3} \times f^{0.16} d^{1.4} + 0.2 \quad (3.50)$$

Constraints are given as:

i) Surface finish

$$SF \leq 50 \ \mu\text{inches} \quad (3.51)$$

where, $$SF = 204.62\text{E}6 \ V^{-1.52} f^{1.004} d^{0.25} \quad (3.52)$$

ii) Horsepower

$$Hp \leq 4 \quad (3.53)$$

where, $$Hp = 2.39 \ V^{0.91} f^{0.78} d^{0.75} \quad (3.54)$$

iii) Feed

$$f \leq 0.1 \ \text{ipr} \quad (3.55)$$

Table 3-7 shows the results of solving this model using SUMT, GINO, BOX and HILL for machining 0.2 inches layer in one pass at four different input vectors. SUMT and GINO gave the same minimum cost of 1.553 dollars/piece with the four input vectors. BOX method provides solutions for the four input vectors but its results are

TABLE 3-7 Results of Solving Ermer and Kromodihardjo's Model with Different Optimization Methods at Four Input Vectors.

Optimization Method	Input Vector		Output			Depth of cut d in	Number of passes n
	speed V fpm	feed f ipr	speed V fpm	feed f ipr	Production cost dollars/piece		
SUMT	320.0	0.0018	433.60	0.0038	1.553	0.2	1
	440.0	0.0018	433.60	0.0038	1.553	0.2	1
	320.0	0.0039	433.60	0.0038	1.553	0.2	1
	440.0	0.0039	433.60	0.0038	1.553	0.2	1
BOX	320.0	0.0018	368.12	0.0066	1.110	0.2	1
	440.0	0.0018	368.12	0.0066	1.110	0.2	1
	320.0	0.0039	345.40	0.0039	1.430	0.2	1
	440.0	0.0039	368.80	0.0066	1.110	0.2	1
HILL	320.0	0.0018	441.04	0.0039	1.555	0.2	1
	440.0	0.0018	438.18	0.0038	1.554	0.2	1
	320.0	0.0039	Input vector infeasible, no solution found				
	440.0	0.0039	Input vector infeasible, no solution found				
GINO	320.0	0.0018	433.60	0.0038	1.553	0.2	1
	440.0	0.0018	433.60	0.0038	1.553	0.2	1
	320.0	0.0039	433.60	0.0038	1.553	0.2	1
	440.0	0.0039	433.60	0.0038	1.553	0.2	1

different from the results of SUMT and GINO. The Table shows that for the first input vector of 320 fpm and 0.0018 ipr the cost by the BOX method was less than that of GINO and SUMT by 28.52%. HILL method failed to find a solution of the model for the first two input vectors but obtained a minimum cost equal to that of GINO and SUMT for the other two input vectors.

3.2.6: Gomtry Algorithm and the Machining Models

The GOMTRY algorithm was tried with of Iwata [9], Petropolous [7], Ermer [10] and Ermer and Kromodihardjo [22] models. In each case it was observed that the program stopped execution after two or three iterations due to exponent underflow problems. Investigations revealed that a matrix obtained during the program execution was ill conditioned and was near singular and therefore its inverse could not be determined. Since the cost models tried with GOMTRY algorithm were solved with GINO, SUMT, HILL and BOX methods it is concluded that this algorithm is not capable of handling the preceding cost optimization models.

3.2.7: Summary of Optimization Methods Performance

It is clear from Tables 3-3 to 3-7 that results given by SUMT and GINO optimization methods are identical for any arbitrary input vector for all the models considered. Furthermore the optimal results are insensitive to any feasible or infeasible input vector for GINO and only slightly sensitive for SUMT. BOX and HILL methods do not accept infeasible starting vectors and show considerable amount of sensitivity of the cost to the input vectors. Hence it is quite obvious that amongst the optimization methods considered both SUMT and GINO have shown better performance with regard to acceptance of feasible or infeasible input vectors and insensitivity to it. This is explained by the fact that both GINO and SUMT are gradient methods while the methods of BOX and HILL are search methods. Moreover the method of BOX uses random numbers between 0 and 1 for the algorithm to proceed.

It can also be seen from Tables 3-3 to 3-7 that the methods of Box and Hill either show greater amount of sensitivity of cost with input vectors or exhibit infeasibility to them for the models by [12], [7], [10], and [22], as compared to the model by [9]. This indicates that the performance of an optimization method also depends upon the model used.

From the above discussion, it is recommended to use SUMT and GINO for solving machining optimization problems.

4. OPTIMIZATION OF MULTIPASS TURNING OPERATIONS

4.1 INTRODUCTION

All machining processes are required to remove a certain amount of material from the workpiece. Whether the machining process is turning, milling or grinding the depth of cut determines the amount of material to be removed. Theoretically it may be desired to take a very deep cut but practically a machining process is constrained by limitations on the machine tool, the cutting tool and the workpiece. For example a given depth of cut may be prohibited either by the capacity of the machine tool or by the required product surface finish. This often leads to removing a given layer of material in a number of passes, so that each pass would take a certain depth of cut which does not violate any of the constraints imposed on the machine-tool-workpiece system. Economic optimization of multipass operations subject to practical constraints provides the optimal values of the machining variables such as feed, speed, depth of cut, as well as the optimal number of passes.

Optimization of multipass machining operations is dependent upon the nature of the objective function and the distribution of the depth of material to be removed amongst a number of passes required to perform the job, subject to a set of operational and design requirements or constraints at each pass. This conditional distribution of the total depth is termed herein as a cutting strategy.

A closer examination of multipass operations may show that several cutting strategies are possible for a certain multipass operation. This is due to the fact that a user may have specific operational or design requirements. For example a specified or required surface finish of a product dictates running of a finishing pass with a surface finish constraint included with the set of constraints of that pass. In this chapter, multipass turning has been taken as a representative case of multipass machining operations to which machining optimization is applied. A review of the multipass turning optimization literature is presented in Section 4.2 with emphasis on the cutting strategies employed by researchers. It is followed by a description of other proposed cutting strategies for multipass turning operations. The proposed strategies are then demonstrated by applying them to Reference [9] model to obtain their optimal cutting variables using SUMT and GINO optimization methods which consistently give the same solution for the respective machining strategies.

4.2 REVIEW OF MULTIPASS TURNING OPTIMIZATION

Table 4-1 shows the objectives and the corresponding types of constraints reported in the literature [5,9,11,12,21,22] for multipass turning optimization. It can be seen from Table 4-1 that the objective of minimum production cost has been used by all authors except [5] ; the objective of minimum production time has been considered by [12] and [5] ; and the objective of maximum profit has been used by [12] only. The Table also indicates that the model constraints fall into two distinct groups: a) the constraints on the upper and lower limits of the cutting variables of feed, speed and depth of cut which are to be determined by optimization and b) the constraints on the process response variables such as power, cutting force, tool life, temperature, machine tool stability and surface finish. As can be seen from Table 4-1, the constraints on the upper and lower limits of speed, feed and depth of cut have been taken into account by almost all the authors. Only Ermer and Kromodihardjo [22] have used a model which does not have a constraint on cutting speed. Iwata, Murotsu and Oba [9] use constraints on all the process response variables except the constraint for temperature at the chip-tool interface. Hati and Rao's model [12] does not account for constraints on machine tool stability and surface finish. Ermer and Kromodihardjo [22] and Lambert and Walvekar [21] have used only the

TABLE 4-1 Objectives and Model Constraints Reported in Literature for Multipass Turning Optimization

Author	Objective			Type of constraint									
	Min cost	Min time	Max profit	V	f	d	Fc	Pc	TL	Temp:	Stab:	SF	Depth Distn:
Iwata [9]	Y	-	-	Y	Y	Y	Y	Y	-	-	Y	Y	$A=\sum_{i=1}^{n} d_i$
Hati & Rao [12]	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	-	-	A=n d
Ermer & Hardjo [22]	Y	-	-	-	Y	Y	-	Y	-	-	-	Y	$A=d_1+d_2$
Unklesbay& Creighton [11]	Y	-	-	Y	Y	Y	Y	Y	-	-	-	Y	#
Lambert & Walvekar [21]	Y	-	-	Y	Y	Y	-	Y	-	-	-	Y	$A=d_1+d_2$
Davis etal [5]	-	Y	-	Y	Y	Y	*	Y	Y	-	-	-	A=n d

#A equals multiples of a small depth
*Tool thrust & spindle torque

Y: Type of objective and constraint used in a particular model
Min: Minimum, Max: Maximum, V: Speed, f: Feed, d: Depth of cut
Fc: Cutting force, Pc: Power, TL: Tool Life
Temp: Tool Temperature, Stab: Stability, SF: Surface finish
Distn: Distribution, A: Total depth of material to be removed

cutting power and surface finish as constraints on process response variables. Unklesbay and Creighton [11] have used cutting force, cutting power and surface finish as constraints. Davis et al [5] has used cutting force and tool life as constraints while taking tool thrust and spindle torque constraints to account for the cutting force constraint.

One of the constraints which is characteristic of multipass machining is the depth distribution constraint which relates the total depth of the layer to be removed to the number of passes 'n' and the respective cutting depths d_i $(i=1,2,\ldots n)$. A number of cutting strategies or methods have been used by researchers to remove a layer of material in a number of passes while meeting certain sets of constraints. Iwata, Murotsu and Oba [9], on optimizing a multipass turning operation have used in addition to other model constraints, the following relationship:

$$\sum_{i=1}^{n} d_i = A \qquad (4.1)$$

where A = depth of layer to be removed and

n = the number of passes required to remove 'A'.

The depth of cut, d_i for the ith pass may or may not be equal to the depth of cut d_j for the jth pass where $i \neq j$. Since the objective considered by [9] is the minimum production cost, the total production cost is the sum of costs for the 'n' passes. The optimal process variables to be determined are the cutting speed, feed rate, number of passes and the depth of cut for each pass.

For multipass turning optimization, Hati and Rao [12] have divided the depth of a material layer to be removed 'A' amongst 'n' equal depth passes according to the equation:

$$n\ d = A \tag{4.2}$$

The constraints are identical for all passes. For a certain value of 'A', the process is optimized at increasing values of 'n', (n=1,2,3...), giving $d = (\frac{A}{n})$. For this case since the depth of cut 'd' is fixed by a given number of passes 'n', feed and speed are the optimization variables to be determined for each integer value of 'n'. The value of 'n', for a given 'A' which results in the lowest production cost is taken as the optimal value. The total production cost is 'n' times the optimal minimum production cost of that pass. Davis etal [5] divided 'A' into 'n' equal passes which satisfy Equation

(4.2) for multipass turning optimization, when minimum production time is the objective.

During the optimization of a two pass turning operation Ermer and Kromodihardjo [22] have used the formulation:

$$d_1 + d_2 = A \tag{4.3}$$

where d_1 is the depth of cut for the roughing pass and d_2 is the depth of cut for the finishing pass. The depth of cut for the roughing pass, d_1, decrements from a value of 'A' to zero. For a given pass only feed and speed are the variables to be obtained by optimization. The total production cost is the sum of costs of the roughing and finishing passes. The purpose of [22] was to show that a suitably selected d_1 and d_2 give a lower production cost for a two pass operation than a single pass operation.

Lambert and Walvekar [21] have also used the formulation of Equation (4.3) for removing the material in two passes. Single pass optimization is carried out for the roughing pass to determine the optimal values of speed V_1, feed f_1, and depth of cut d_1. For the finishing pass, since the depth of cut $d_2 = A-d_1$ is known, only the optimal values of speed V_2, and feed f_2 as variables are determined. The total production cost for [21] is the sum of the roughing and finishing passes costs. The constraints of the finishing pass are different from those of the roughing pass by including a constraint on the surface finish.

Thus for multipass turning optimization, Equations (4.1) and (4.2) represent two distinct strategies for removing 'A'. The effect of choosing or using these machining strategies on Reference [9] objective function and set of constraints, listed in Tables 4-2 and 4-3 respectively, is observed from their respective optimal results. The optimal cost and process variables to remove material layers of A=4,6,8 and 10 mm thickness using the two depth strategies are shown in Table 4-4. It is evident from Table 4-4 that the minimum production costs for **n d** = A depth distribution constraint, solved by GINO and SUMT are 15%, 11%, 10% and 9.5% less expensive than those given by [9] for A= 4,6,8 and 10 mm respectively. The optimal number of passes and minimum cost are shown plotted against the total depth of the layer to be removed, 'A', in Fig. 4-1 and Fig. 4-2 respectively for these two cutting strategies. The Figures show that although the Reference [9] strategy consistently uses one pass less than the A=nd strategy to remove a given layer of material, the

TABLE 4-2 Objective Function and Constraints for Reference [9]

Objective:

Cost/pass = Machining Cost + Tooling Cost + Handling Cost

$$= C_o t_m + \left(\frac{t_m}{TL}\right)(C_o t_e + C_t) + C_o t_p$$

where Co = Operating cost, t_m = Machining time ,
TL = Tool life, t_e = Tool changing time
C_t = Tool cost per cutting edge, t_p = Handling time

Total Cost:

$$= \sum_{i=1}^{n} 3927 V_i^{-1} f_i^{-1} + 1.95E\text{-}8 V_i^{2.887} f_i^{-1} e^{5.884 \cdot f_i} d_i^{-1.117} + 10$$

where 'n' is the number of passes

Constraints:

1) Cutting speed

$$14.13 \le V_i \le 1005.3 \quad \text{m/min}$$

2) Feed

$$0.01 \le f_i \le 5.6 \quad \text{mm/rev}$$

3) Depth of cut

$$0 < d_i \le A \text{ mm}$$

4) Cutting force

$$290.73\ V_i^{-0.1013}\ f_i^{0.725}\ d_i \le 170 \text{ kg}$$

5) Power

$$(290.73\ V_i^{-0.1013}\ f_i^{0.725}\ d_i)\ \frac{V_i}{4896} \le 7.5 \text{ kW}$$

6) Stability

$$f_i\ V_i^2 \ge 2230.5$$

7) Surface finish

$$0.356\ f_i^2 \le 0.06 \text{ mm}$$

8) Depth distribution

$$\sum_{i=1}^{n} d_i = A$$

TABLE 4-3 Objective Function and Constraints for A=nd Approach

Objective:

Cost/pass = Machining Cost + Tooling Cost + Handling Cost

$$= C_o t_m + \left(\frac{t_m}{TL}\right)(C_o t_e + C_t) + C_o t_p$$

where Co = Operating cost, t_m = Machining time ,
TL = Tool life, t_e = tool changing time
C_t = Tool cost per cutting edge, t_p = Handling time

Total Cost:

$$= n(\ 3927V^{-1}f^{-1} + 1.95E\text{-}8V^{2.887}f^{-1}\ e^{5.884 \cdot f}d^{-1.117} + 10)$$

where 'n' is the number of passes

Constraints:

1) Cutting speed

$$14.13 \leq V \leq 1005.3 \quad m/min$$

2) Feed

$$0.01 \leq f \leq 5.6 \quad mm/rev$$

3) Depth of cut

$$0 < d \leq A \ mm$$

4) Cutting force

$$290.73\ V^{-0.1013}\ f^{0.725}\ d \leq 170\ kg$$

5) Power

$$(290.73\ V^{-0.1013}\ f^{0.725}\ d)\ \frac{V}{4896} \leq 7.5\ kW$$

6) Stability

$$f\ V^2 \geq 2230.5$$

7) Surface finish

$$0.356\ f^2 \leq 0.06\ mm$$

8) Depth distribution

$$n\ d = A$$

TABLE 4-4 Comparison of Results for Iwata, Murotsu and Oba's Approach [9] with the A=nd Approach

A	Iwata, Murotsu and Oba [9]					A=nd approach					Difference
mm	n	V m/min	f mm/rev	d mm	Cost yens/pc	n	V	f	d	Cost	in cost over[9]
4	1	216.00	0.1467	4.00	195.00	2	216.00	0.3886	2.00	165.88	15%
							216.00	0.3886	2.00		
6	2	216.00	0.2189	3.00	250.70	3	216.0	0.3886	2.00	224.09	11%
		216.00	0.2189	3.00			216.0	0.3886	2.00		
							216.0	0.3886	2.00		
8	3	216.0	0.3828	2.00	314.34	4	216.0	0.3886	2.00	282.12	10%
		216.0	0.2189	3.00			216.0	0.3886	2.00		
		216.0	0.2189	3.00			216.0	0.3886	2.00		
							216.0	0.3886	2.00		
10	4	216.0	0.3828	2.00	375.60	5	216.0	0.3886	2.00	340.15	9.5%
		216.0	0.3828	2.00			216.0	0.3886	2.00		
		216.0	0.2189	3.00			216.0	0.3886	2.00		
		216.0	0.2189	3.00			216.0	0.3886	2.00		
							216.0	0.3886	2.00		

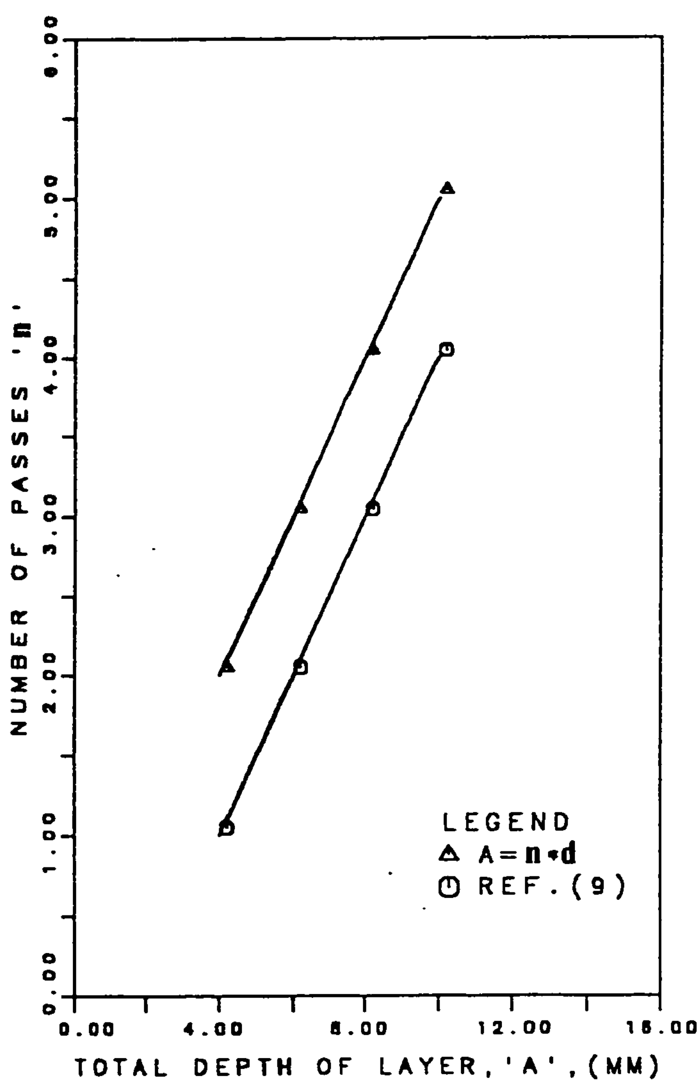

Figure 4-1 Optimal number of passes required to remove a given depth of layer using two machining strategies.

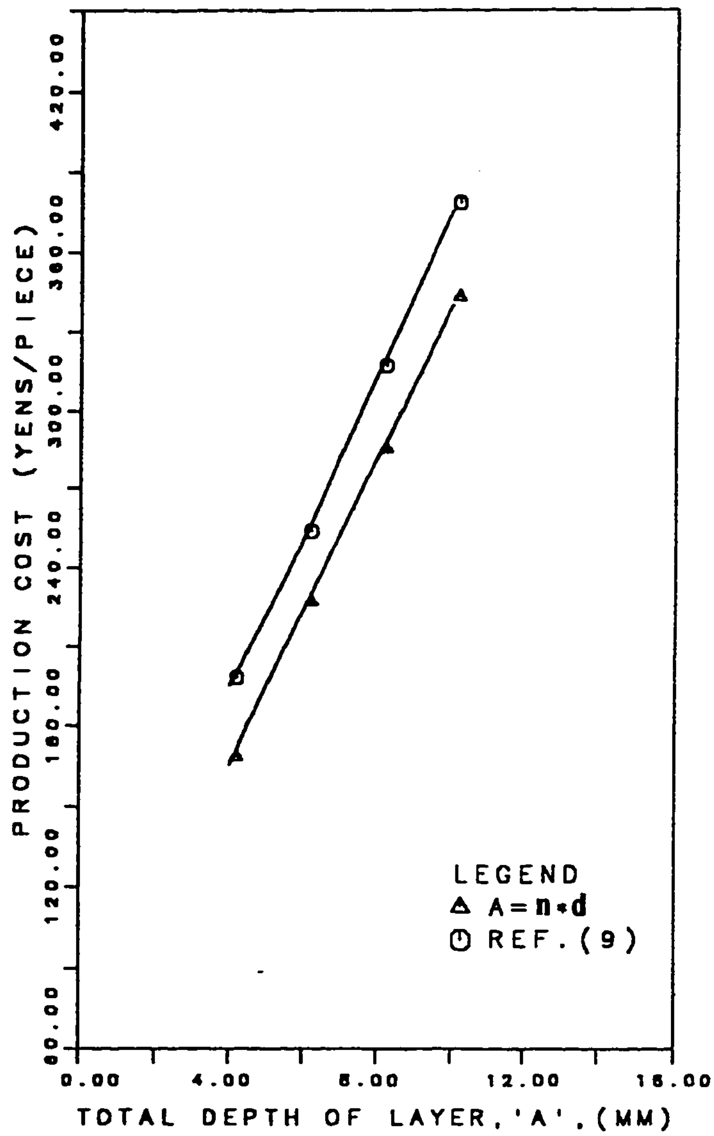

Figure 4-2 Optimal production cost for removing a given depth of layer using two machining strategies.

production cost for the former strategy is less than that of the latter. For example to remove a layer of material A=8 mm deep, Fig. 4-1 shows that when the depth distribution constraint nd=A is used, the optimal number of passes is 4; while the optimal number of passes for [9] is 3. However from Fig. 4-2 the corresponding production cost for A=8 mm for A=nd approach is 282.12 yens/piece as compared to 314.34 yens/piece for [9]. Thus just by changing the selection from the cutting strategy of [9] to A=nd strategy with the objective function of cost and constraint set of [9], different optimal minimum production costs and the optimal number of passes are obtained. This points to the fact that the results of optimization are dependent on the cutting strategy employed.

Variations in the operational, design or end user requirements can lead to a number of possible cutting strategies which include but are not limited to the two above mentioned strategies. The following section proposes five cutting strategies which can be suitable for multipass turning operations optimization. These strategies will be found to differ in the method by which the total depth of cut is divided into passes while satisfying respective sets of constraints usually dictated by operational, design or end user's requirements. The proposed strategies are demonstrated by applying them to the minimum cost objective function and the corresponding constraints of

[9]. The optimal minimum cost and process variables of the proposed strategies, obtained by GINO or SUMT optimization methods, are compared and evaluated to determine the respective merits of the strategies. Although the proposed strategies are demonstrated by the minimum production cost as the optimization objective, they are equally applicable for minimum production time, maximum material removal rate or maximum profit objectives.

4.3 PROPOSED MULTIPASS CUTTING STRATEGIES

Based on a variety of operational, design and end user requirements the following scenarios or cutting strategies are proposed for multipass turning operations. These strategies can be compared to the individual intuitive approaches that a number of engineers would independently pursue to optimally remove a layer of material 'A' from a workpiece while satisfying an objective (e.g. minimum cost, etc.) subject to a set of operational constraints.

4.3.1: Multipass Strategy 1 (MS1)

This cutting strategy is based on removing a layer of material of depth 'A' in an integral number of equal depth passes,'n', which satisfy the following depth constraint:

$$n\,d = A \tag{4.4}$$

where 'n' is the integral number of passes and 'd' is the depth of cut at each pass and all passes are subject to the same set of constraints. The distribution of the machining passes to remove the depth 'A' from a bar using MS1 is shown schematically in Fig. 4-3(a).

The minimum production cost for the process is 'n' times the cost of the optimal single pass. The objective function to be minimized is given by:

$$C_T = n \times C_p \tag{4.5}$$

where C_T = total production cost

C_p = production cost of a single pass, which is function of the machining variables of speed 'V', feed rate 'f', and depth of cut 'd' such as that given in Tables 4-3 and 4-4. The optimal minimum production cost of this strategy is obtained for increasing values of n=1,2,3,.., by an optimization process. The value of 'n' which results in the lowest production cost is taken as the optimal number of passes 'n', and the values of 'V', 'f' and d=A/n are the corresponding optimal process variables for the MS1 strategy.

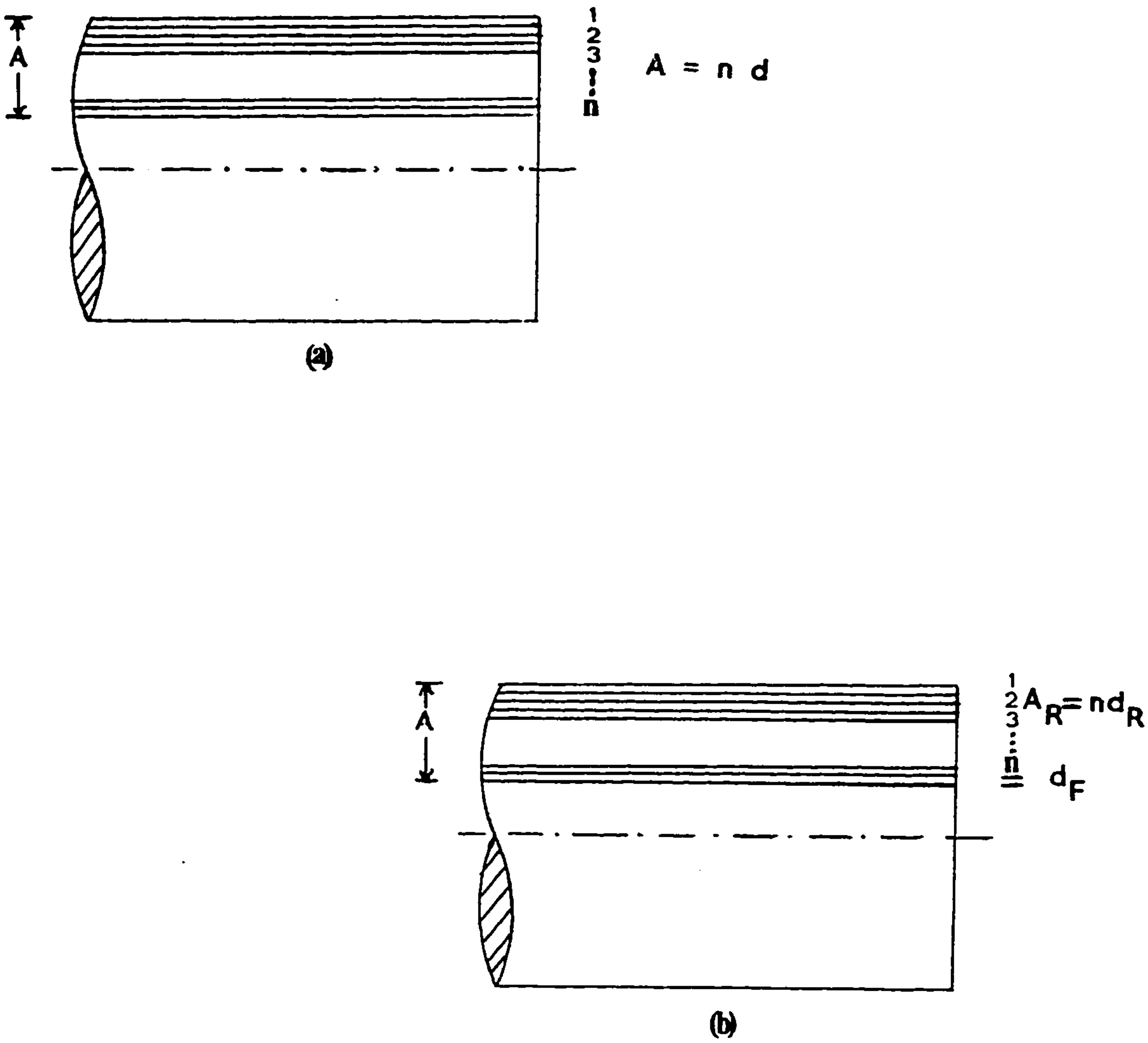

Figure 4-3 Depth distribution for:
(a) Multipass Strategy 1 (MS1)
(b) Multipass Strategy 2 (MS2)

4.3.2: Multipass Strategy 2 (MS2)

This machining strategy becomes necessary when a layer of material of total depth 'A' has to be removed in several passes which include a final finishing pass with a required level of surface finish for the workpiece. The optimal minimum production cost and the corresponding optimal roughing speed V_R, feed f_R, and depth of cut d_R are first determined for a single pass roughing operation named the optimal roughing pass. The constraint on surface finish for the roughing pass is either removed or given a relatively large value. The other constraints may remain the same for both the roughing and finishing passes. The number of roughing passes 'n' is chosen so that the total depth for roughing, $A_R = nd_R$ nears the value of the total depth to be removed 'A', and an optimal finishing pass is then determined for the balance of the material layer. The finishing pass is optimized subject to the depth of cut constraint $d_F = A - A_R$ and an additional surface finish constraint. The total production cost for the strategy is thus given by 'n' times the cost of the optimal roughing pass plus the optimal cost of the finishing pass. The depth distribution for MS2 is shown schematically in Fig. 4-3(b).

When the optimal roughing depth of cut d_R is of such a value so that $A = nd_R$, then the material is removed in (n-1) optimal roughing passes and one finishing pass. The finishing pass is optimized using a depth of cut, $d_F = d_R$, to obtain the optimal finishing pass cost and the corresponding values of cutting speed, V_F and feed, f_F. The total production cost is given by (n-1) times the cost of the optimal roughing pass plus the cost of the optimal finishing pass.

4.3.3: Multipass Strategy 3 (MS3)

This strategy is similar to MS2 in Section 4.3.2, where the layer of depth 'A' has to be removed in 'n' roughing passes and one finishing pass. However, the optimal finishing pass conditions are determined before those of the optimal roughing pass. The optimal minimum production cost is first obtained for a single pass finishing cut. The surface finish constraint is included in the set of constraints for this pass. The optimal depth of cut obtained for finishing d_F is subtracted from the total depth 'A', to obtain the total depth $A_R = A - d_F$ for the roughing passes, which is to be removed in $n = \frac{A_R}{d_R}$ identical and optimal roughing passes. The optimal minimum roughing pass cost is obtained at increasing values of n=1,2,3,.. (giving $d_R = \frac{A_R}{n}$ at each 'n') and the value of 'n' that results in the lowest minimum production cost gives the optimal roughing pass process variables V_R, f_R and d_R. The surface finish constraint is either discarded or given a very high value for optimizing the roughing pass. The total production cost is thus given by the cost of the 'n' optimal roughing passes plus the cost of one optimal finishing pass.

The depth distribution for MS3 is shown schematically in Fig. 4-4(a).

4.3.4: The Unified Multipass Strategy (MS4)

This machining strategy consists of making 'n' identical optimal roughing passes and one final finishing pass. The total depth to be removed satisfies the relationship:

$$n\, d_R + d_F = A \tag{4.6}$$

where n = 0,1,2,3..., is the number of roughing passes,

d_R = depth of the roughing pass and

d_F = depth of the finishing pass.

Subscripts R and F stand for roughing and finishing respectively.

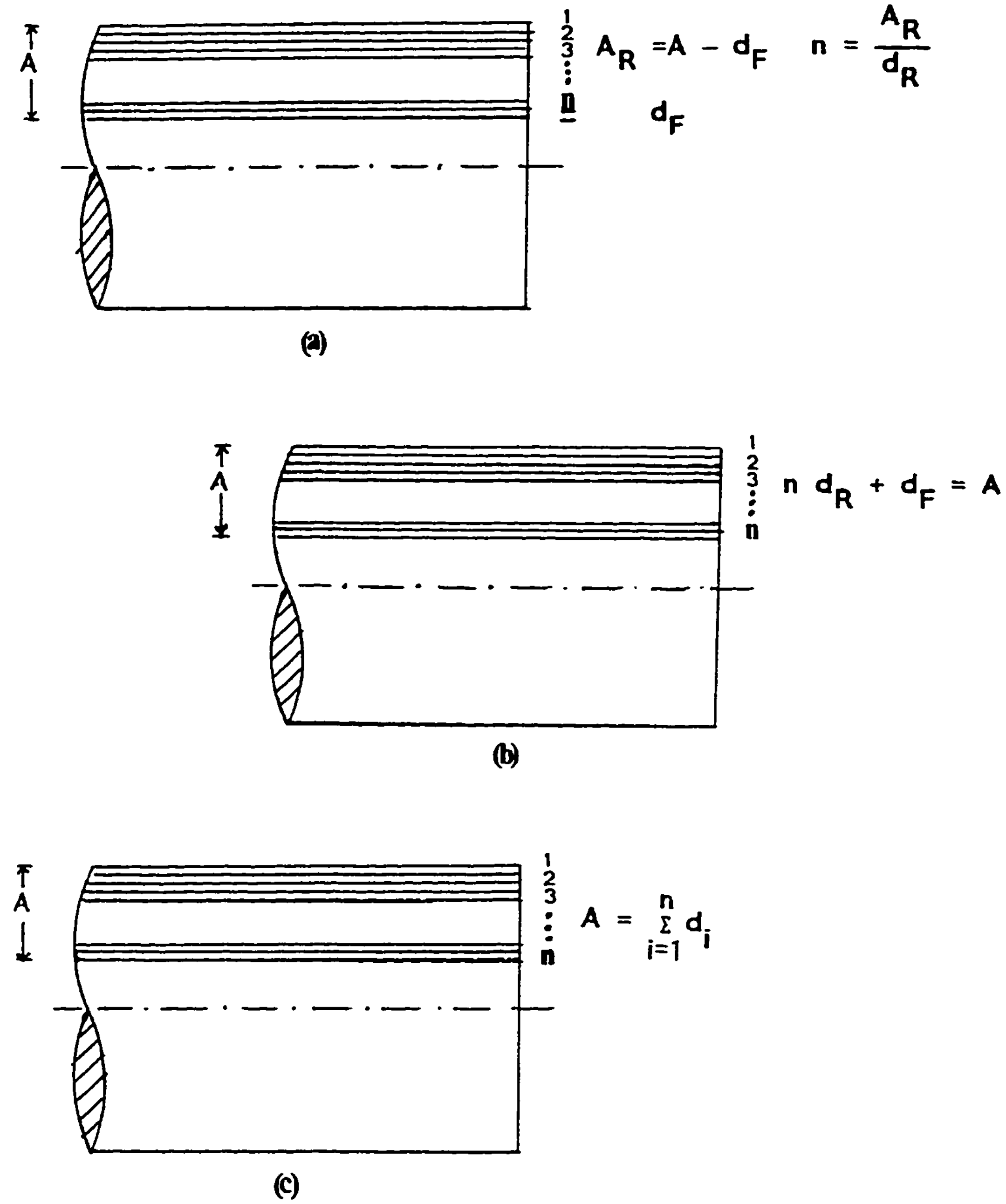

Figure 4-4 Depth distribution for:
(a) Multipass Strategy 3 (MS3)
(b) Unified Multipass Strategy (MS4)
(c) Generalized Multipass Strategy (MS5)

The depth distribution constraint is shown schematically in Fig. 4-4(b).

The optimization of the roughing and finishing passes of this machining strategy is carried out simultaneously. This is contrary to the approaches of Sections 4.3.2 and 4.3.3 where the roughing passes and finishing pass optimization are performed separately. The total production cost for this strategy is a function of the machining variables of both the roughing and finishing passes $V_R, f_R, d_R, V_F, f_F, d_F$, where subscripts R and F correspond to roughing and finishing passes respectively. The total production cost is 'n' times the cost of the roughing pass plus the cost of the finishing pass. For example, the objective of the minimum production cost of reference [9] for this strategy is given as follows:

$$\text{Cost} = n \times \text{roughing pass cost} + \text{finishing pass cost} \qquad (4.7)$$

where:

Roughing pass cost

$$= n\,(3927\, V_R^{-1}\, f_R^{-1} + 1.95E\text{-}8\, V_R^{2.887}\, f_R^{-1}\, e^{5.884 \cdot f_R}\, d_R^{-1.117} + 10) \qquad (4.8)$$

and Finishing pass cost

$$= (3927\ V_F^{-1}\ f_F^{-1} + 1.95E\text{-}8\ V_F^{2.887}\ f_F^{-1}\ e^{5.884 \cdot f_F}\ d_F^{-1.117} + 10) \tag{4.9}$$

During the optimization using this strategy two sets of constraints are in effect: one set is for the roughing and the other set is for the finishing pass. The depth distribution constraint of Equation (4.6) relating A, d_R and d_F is included with the constraints. The empirical equations of the first set of constraints are functions of the roughing pass variables (V_R, f_R, d_R) whereas the second set of constraints are function of the finishing pass variables (V_F, f_F, d_F) and also includes the surface finish constraint. For a given total depth 'A', the objective function is optimized and optimal costs are compared for successive values of 'n'. The least value of the cost obtained for a value of 'n' is taken as the optimal minimum and the corresponding values of the machining variables are recorded, i.e. $(n, V_R, f_R, d_R, V_F, f_F, d_F)$. This strategy thus unifies both roughing and finishing operations and is therefore termed the unified strategy, where roughing is performed in an integral number of passes while the last pass is the finishing pass.

4.3.5: The Generalized Multipass Strategy 5 (MS5)

In this machining strategy the total depth 'A' is removed in 'n' possibly independent passes which have to be optimized in one step. The total objective is the sum of the individual costs of the 'n' passes. The value of the objective is the sum of the objectives of the 'n' passes e.g.

$$\text{Total production cost} = cost_1 + cost_2 + \ldots + cost_i + \ldots + cost_n \qquad (4.10)$$

where $cost_i$ = cost of the ith pass.

Each pass may have a different set of constraints dictated by the operational, design and end user requirements. The last pass may have a surface finish constraint in addition to other constraints.

The depth of cuts of the 'n' passes have to satisfy the depth disstribution equality constraint:

$$\sum_{i=1}^{n} d_i = A \qquad (4.11)$$

where 'n' is the optimal number of passes for the operation. Fig. 4-4(c) shows the depth distribution constraint schematically.

For MS5 strategy the process is optimized at increasing values of $n=1,2,3,\ldots$ and the value of 'n' which results in the least production cost is the optimal number of passes and corresponding cost is the optimal cost. The number of search variables of the strategy depends upon the value of 'n' because each pass has a set of process variables and a corresponding set of constraints which have to be obtained. For example for n=3, there are nine variables; speed V_i, feed f_i, and depth of cut d_i for each pass i.e. $i=1,2,3$. This cutting strategy seems to be similar to the one given in [9]. The only difference is that in [9], the number of passes 'n' is a search variable within the optimizing algorithm, while for MS5, integral values of 'n' are fed into the optimizing algorithm to obtain the optimum value of the objective function and the corresponding variables.

It can be seen that strategy MS1 is a special case of MS5. This can be accomplished by putting $d_i=d$ in Equation (4.11) which results in $\sum_{i=1}^{n} d_i = n\,d = A$ or Equation (4.4) of MS1. On the other hand, if (n-1) passes have identical sets of constraints and the last pass has a different set of constraints, it is conjectured that strategy MS5 reduces to strategy MS4.

4.4 PERFORMANCE EVALUATION OF THE CUTTING STRATEGIES

In this section the model of multipass turning of mild steel workpiece with a carbide cutting tool reported by Iwata, Murotsu and Oba [9] has been selected to demonstrate and compare the performance of the proposed multipass turning strategies. The optimization methods of SUMT and GINO have been used for solving the problem for each strategy.

The following minimum production cost objectives and constraints are used for the respective cutting strategies.

a) For strategy MS1:

The production cost per piece is given as:

$$\text{Cost} = n \times \text{cost of an optimal machining pass} \tag{4.12}$$

$$\text{Cost} = n(3927\ V^{-1}\ f^{-1} + 1.95E\text{-}8\ V^{2.887}\ f^{-1}\ e^{5.884 \cdot f}\ d^{-1.117} + 10) \tag{4.13}$$

The model constraints are given by Equations (4.28) to (4.36) and the following depth distribution constraint:

$$A = n \times d \tag{4.14}$$

b) For strategy MS2:

The cost of single pass roughing which is optimized in the first stage is given by:

$$Cost_R = 3927\ V_R^{-1}\ f_R^{-1} + 1.95E\text{-}8\ V_R^{2.887}\ f_R^{-1}\ e^{5.884 \cdot f_R}\ d_R^{-1.117} + 10 \tag{4.15}$$

Although no depth distribution constraint is used for the roughing pass, the constraints of Equations (4.28) to (4.36) apply to this stage of optimization.

The cost of finishing pass which is optimized in the second stage is given by:

$$Cost_F = 3927\ V_F^{-1}\ f_F^{-1} + 1.95E\text{-}8\ V_F^{2.887}\ f_F^{-1}\ e^{5.884 \cdot f_F}\ d_F^{-1.117} + 10 \tag{4.16}$$

The cost has to be minimized subject to the Equations (4.28) to (4.36) constraints and to the depth distribution constraint given by:

$$d_F = A - n\ d_R \tag{4.17}$$

where 'n' is the number of optimal roughing passes. Thus the total number of passes for the operation is 'n+1'.

c) For strategy MS3:

The production cost for the optimal single pass finishing of the first stage optimization is given by:

$$\mathrm{Cost}_F = 3927\ V_F^{-1}\ f_F^{-1} + 1.95E\text{-}8\ V_F^{2.887}\ f_F^{-1}\ e^{5.884 \cdot f_F}\ d_F^{-1.117} + 10 \tag{4.18}$$

Since no depth distribution constraint is imposed for the finishing pass, only the constraints in Equations (4.28) to (4.36) are used.

The production cost for the 'n' roughing passes which are optimized in a second stage is given by:

$$\mathrm{Cost} = n \times \text{optimal roughing pass cost} \tag{4.19}$$

$$\mathrm{Cost}_R = n(3927\ V_R^{-1}\ f_R^{-1} + 1.95E\text{-}8\ V_R^{2.887}\ f_R^{-1}\ e^{5.884 \cdot f_R}\ d_R^{-1.117} + 10) \tag{4.20}$$

The model constraints are those of Equations (4.28) to (4.36) and the depth distribution constraint is given by:

$$A_R = n\ d_R \tag{4.21}$$

where,

$$A_R = A - d_F. \tag{4.22}$$

The total number of passes for the operation is the sum of the number of optimal roughing passes 'n' and one finishing pass.

d) For strategy MS4:

The production cost is given by:

$$\text{Cost} = n \times \text{roughing pass cost} + \text{finishing pass cost} \tag{4.23}$$

$$= n(3927\ V_R^{-1}\ f_R^{-1} + 1.95E\text{-}8\ V_R^{2.887}\ f_R^{-1}\ e^{5.884 \cdot f_R}\ d_R^{-1.117} + 10)$$

$$+\ 3927\ V_F^{-1}\ f_F^{-1} + 1.95E\text{-}8\ V_F^{2.887}\ f_F^{-1}\ e^{5.884 \cdot f_F}\ d_F^{-1.117} + 10 \tag{4.24}$$

The constraints of Equations (4.28) to (4.36) are applied to the roughing and the finishing passes. The depth distribution constraint is given by

$$n\ d_R + d_F = A \tag{4.25}$$

e) For strategy MS5:

The production cost is given by:

$$\text{Cost} = \sum_{i=1}^{n} 3927\ V_i^{-1}\ f_i^{-1} + 1.95E\text{-}8\ V_i^{2.887}\ f_i^{-1}\ e^{5.884 \cdot f_i}\ d_i^{-1.117} + 10 \tag{4.26}$$

The depth distribution constraint is given by:

$$\sum_{i=1}^{n} d_i = A \tag{4.27}$$

where 'n' is the number of passes.

The constraints other than that of the depth distribution which are common to all the strategies are as follows:

i) The maximum and minimum cutting speeds

$$14.13 \le V \le 1005.3 \quad \text{m/min} \tag{4.28}$$

ii) The maximum and minimum feeds

$$0.01 \le f \le 5.6 \quad \text{mm/rev} \tag{4.29}$$

iii) The maximum and minimum depth of cut

$$0 < d \le A \text{ mm} \tag{4.30}$$

where 'A' is the total depth of material to be removed. In this section A=10 mm.

vi) The maximum cutting force

$$F_c \le 170 \text{ kg} \tag{4.31}$$

where, $$F_c = 290.73\ V^{-0.1013}\ f^{0.725}\ d \tag{4.32}$$

vii) The maximum power consumption

$$P_c \le 7.5 \text{ kW} \tag{4.33}$$

where, $$P_c = \frac{F_c\ V}{4896} \tag{4.34}$$

vi) The stable cutting region related to the cutting surface

$$f\ V^2 \ge 2230.5 \tag{4.35}$$

vii) The limited maximum surface roughness

$$0.356\ f^2 \le H_{max} \tag{4.36}$$

H_{max} is being taken as 0.06 mm for the roughing passes and 0.01 mm for the finishing pass.

For a total depth of 10 mm, GINO has been used to solve the problem corresponding to strategies MS1 to MS5 and results tabulated in Table 4-5. The Table also includes the results given by the strategy of [9]. From the Table it can be seen that while strategy MS2 incurs the highest production cost of 453.46 yens/piece, strategies MS4 and MS5 incur the least and equal production costs which are 22.72% cheaper than strategy MS2. Strategy MS3 provides a cost of 359.9 yens/piece, an improvement of 20.6% over the cost for strategy MS2. Strategy MS1 gives a cost which is cheaper than strategy MS2 by 17.6%. The result given by [9] is 16.3% less expensive than strategy MS2 but 8.2% more expensive than strategies MS4 and MS5 and 5.4% more expensive than strategy MS3. The result given by [9] differs from the results of the proposed strategies because [9] employs a different strategy alongwith its own optimization algorithm (Section 4.2).

The equality of cost for strategy MS4 and MS5 is conjectured to be attributed to the MS5 reducing to MS4 when (n-1) roughing passes and one final finishing pass is used to remove 'A', which is the case in this example. Strategy MS1 requires all passes to have

TABLE 4-5 Comparison of the Results for the Different Cutting Strategies, A=10 mm

	MS1	MS2	MS3	MS4	MS5	Results by [9]
Passes n_R, n_F	-,3	7,1	4,1	4,1	4,1	2,1
Depths of cut d_R, d_F mm	-,3.33	1.31,0.84	2.10,1.68	1.92,2.31	1.92,2.33	3.0,4.0
Speed V_R, V_F m/min	-,240.86	331.2,454.3	216,514	216,362	216,363	216,216
Feed f_R, f_F mm/rev	-,0.168	0.411,0.168	0.368,0.168	0.411,0.167	0.411,0.167	0.219,0.147
Production cost yens/piece	373.20	453.46	359.90	350.42	350.68	379.40

Subscripts R and F denote roughing and finishing respectively identical constraints. To meet the finishing requirement of the final pass, all passes were taken as finishing passes. The total cost is 'n' times the optimal finishing pass cost. This contributes to the higher cost for strategy MS1 as compared to strategy MS4 or MS5. For strategy MS4 the depth distribution constraint, $A = n\ d_R + d_F$, reduces to the depth distribution constraint of MS1, i.e. A=nd, if for MS4 the last pass for finishing is eliminated or all the passes are taken at identical conditions.

The 22.7% difference in production costs for A=10 mm between strategies MS2 and MS4 can be explained by the fact that MS2 strategy is optimized in two stages whereas MS4 is optimized in one stage only. For MS2 an optimal roughing pass satisfying the roughing pass constraints is first obtained. The cutting is assumed with the same roughing conditions until most of the material is removed in 'n' integral passes. The balance is then removed in a final finishing pass which satisfies a smooth surface finish constraint. The cost is 'n' times the optimal roughing pass cost plus the cost of the finishing pass. However in strategy MS4, the model is optimized in one step using the combined cost of finishing and roughing (Equation (4.21)). and the depth distribution constraint of $A = n\ d_R + d_F$.

The results of the preceding paragraphs have shown that for the same model constraints and a given value of total depth 'A', MS1 to MS5 give different minimum cost results. In the following section a model similar to [9] with maximum number of relevant constraints is considered and solved for each of the cutting strategies MS1 to MS5 under selected machining conditions using GINO and SUMT optimization methods. The objective of the study is to investigate the relative sensitivity of the proposed strategies to the machining process variables.

4.5 SENSITIVITY ANALYSIS OF THE PROPOSED CUTTING STRATEGIES

In this section, the proposed cutting strategies MS1 to MS5 are applied to a general multipass turning operation model and optimized for selected values of 'A', selected restrictions on surface finish and tool life constraints. The results of this study are evaluated to asses the relative performance and sensitivity of the proposed strategies to some of the process variables and constraints which have direct relationship to the design requirements of the part and the economics of the process. The machining model to which MS1 to MS5 strategies are applied is first presented in Section 4.5.1 followed by the optimization results of the strategies and the evaluation of their sensitivity to surface finish, to tool life and to total depth of material to be removed,'A', in Sections 4.5.2, 4.5.3 and 4.5.4 respectively.

4.5.1: A General Machining Optimization Model

A general optimization model for multipass turning operation which incorporates most operational, design and end user requirements is selected for testing the sensitivity of the preceding cutting strategies. The general model is assumed to incorporate, in addition to the objective function, several important operational and end user constraints. Each of the constraints has to be represented by an empirical equation with coeffecients dependent on the machine tool, cutting tool and the workpiece material used for a multipass operation. The general model presents to the user the opportunity to select a set of constraints, from the total number of constraints, which are of a specific interest for his application and drop the ones which are of less or no interest. For multipass turning operation, a general model is assumed to have ten constraints, namely: constraints for speed, feed, depth of cut, power, cutting force, process stability, temperature, tool life, surface finish and depth distribution.

To make the model by [9] more general, its set of constraints has been supplemented by a temperature constraint borrowed from Reference [12] and the theoretical surface finish constraint given by [9] has been replaced by an empirical relationship taken from [38]. Although the objective function of [9] (Table 4-2) contains the expression of tool life in terms of the machining variables (V,f,and d), the tool life was not restricted by assigned values. Hati and Rao [12] and Davis et al [5] have used tool life constraint values for the multipass turning operations. The restriction on tool life becomes essential when a tool is expected to perform well for a certain period of time after which it should be sharpened or replaced. This is an important factor especially with the use of planned tool replacement strategies in modern numerical control machine tools and machining centers. Therefore to make the model of [9] a general model the set of constraints given by Equations (4.28) to (4.35) are used along with the following constraints.

Temperature constraint:

$$T = 132\ V^{0.4}\ f^{0.2}\ d^{0.105} \tag{4.37}$$

$$T \leq 1000°C \tag{4.38}$$

Surface finish, centerline average constraint:

$$SF = 141.65\ V^{-0.587}\ f^{1.179}\ d^{0.165} \tag{4.39}$$

Tool life constraint:

$$TL = 4.029E11\ V^{-3.887}\ e^{-5.884 \times f}\ d^{1.117} \tag{4.40}$$

The following sections present the optimization results of MS1 to MS5 and the evaluation of the sensitivity to variations in finishing pass surface finish, tool life and total layer of material 'A'.

4.5.2: Sensitivity of the Strategies to Surface Finish Constraint in the Final Pass

In order to determine the sensitivity of the proposed strategies to surface finish constraints, the general model has been solved for MS1 to MS5, while machining a layer of material of total depth of A=2,4,5,6,7,8,10,15,16,17,18,19,20 and 25 mm, using two levels of surface finish constraints in the final finishing pass. The centerline average of these levels are as follows:

Level 1 : $SF_F \leq 2\mu meters$ (4.41)

Level 2 : $SF_F \leq 1\mu meter$ (4.42)

The objective functions of the cost per piece and depth distribution constraints corresponding to MS1 to MS5 are given by Equations (4.12) to (4.27) respectively. The other constraints which are common to all strategies are given by Equations (4.28) to (4.35) and (4.37) to (4.40). The surface finish is restricted to a centerline average value of 6 μmeters in the roughing passes, while the tool life is constrained between 1 and 200 mins for both the roughing and the finishing passes, i.e. $1 \leq TL \leq 200$.

Tables 4-6 to 4-13 list the detailed results of optimization of the strategies for multipass machining for A=2,4,5,6,7,8,10 and 15 mm,

TABLE 4-6 (a) Optimal Cost and Process Variables for MS1 to MS5 for A=2 mm
$SF_F \leq 2\ \mu m, SF_R \leq 6\ \mu m$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	45.14	45.25	--	--
$Cost_F$ yens/piece	60.98	51.99	51.95	60.98	60.96
SF_F μm	1.99	2.00	2.00	2.00	1.99
Passes n_R, n_F	-,1	1,1	1,1	-,1	-,1
Speed V_R, V_F m/min	-,219.88	171.28,252.18	173.60,254.20	-,219.83	-,219.79
Feed f_R, f_F mm/rev	-,0.358	0.780,0.435	0.745,0.439	-,0.358	-,0.359
Depth of cut d_R, d_F mm	-,2.0	1.178,0.822	1.22,0.78	-,2.0	-,2.0
Tool life TL_R, TL_F mins	-,83.15	10.18,11.54	12.31,10.26	-,83.15	-,83.06
Total Cost	60.98	97.13	97.20	60.98	60.96

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-6 (b) Optimal Cost and Process Variables for MS1 to MS5 for A=2 mm
$SF_F \leq 1$ µm, $SF_R \leq 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	45.144	45.72	--	--
$Cost_F$ yens/piece	74.71	62.78	62.50	74.71	74.72
SF_F µm	1.00	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,1	1,1	1,1	-,1	-,1
Speed V_R, V_F m/min	-,278.23	171.28,319.10	177.02,325.76	-,278.23	-,278.20
Feed f_R, f_F mm/rev	-,0.224	0.780,0.271	0.699,0.279	-,0.224	-,0.224
Depth of cut d_R, d_F mm	-,2.0	1.178,0.822	1.28,0.72	-,2.0	-,1.999
Tool life TL_R, TL_F mins	-,73.56	10.18,12.08	15.79,9.16	-,73.57	-,73.55
Total Cost	74.71	107.92	108.22	74.71	74.72

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-7 (a) Optimal Cost and Process Variables for MS1 to MS5 for A=4 mm
$SF_F \leq 2$ µm, $SF_R \leq 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	135.43	102.13	56.48	56.48
$Cost_F$ yens/piece	121.97	56.38	51.95	61.98	62.02
SF_F µm	1.99	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,2	3,1	2,1	1,1	1,1
Speed V_R, V_F m/min	-,219.88	171.28,251.27	193.90,254.20	206.24,217.86	206.50,217.80
Feed f_R, f_F mm/rev	-,0.358	0.780,0.470	0.516,0.439	0.421,0.354	0.421,0.354
Depth of cut d_R, d_F mm	-,2.0	1.178,0.466	1.61,0.78	1.87,2.12	1.87,2.12
Tool life TL_R, TL_F mins	-,83.15	10.18,5.02	41.99,10.26	68.64,94.69	68.24,95.12
Total Cost	121.97	191.81	154.08	118.46	118.50

SF: Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-7 (b) Optimal Cost and Process Variables for MS1 to MS5 for A=4 mm
$SF_F \le 1$ µm, $SF_R \le 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	135.43	103.30	48.07	48.06
$Cost_F$ yens/piece	145.13	66.32	62.50	80.12	80.16
SF_F µm	0.36	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,1	3,1	2,1	1,1	1,1
Speed V_R, V_F m/min	-,320.80	171.28,336.51	195.30,325.76	185.91,267.95	185.99,267.90
Feed f_R, f_F mm/rev	-,0.09	0.780,0.301	0.503,0.279	0.594,0.212	0.594,0.212
Depth of cut d_R, d_F mm	-,4.0	1.178,0.466	1.64,0.72	1.44,2.55	1.44,2.55
Tool life TL_R, TL_F mins	-,199.99	10.18,4.360	44.83,9.16	27.77,119.69	27.56,120.01
Total Cost	145.13	201.75	165.80	128.19	128.22

SF: Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-8 (a) Optimal Cost and Process Variables for MS1 to MS5 for A=5 mm
$SF_F \le 2$ µm, $SF_R \le 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	180.57	122.28	74.35	74.50
$Cost_F$ yens/piece	148.73	63.71	51.95	74.37	74.19
SF_F µm	1.60	2.00	2.00	1.61	1.61
Passes n_R, n_F	-,2	4,1	2,1	1,1	1,1
Speed V_R, V_F m/min	-,216.00	171.28,217.52	216.00,254.20	216.00,216.00	216.00,216.00
Feed f_R, f_F mm/rev	-,0.285	0.780,0.469	0.361,0.439	0.285,0.285	0.285,0.285
Depth of cut d_R, d_F mm	-,2.5	1.178,0.288	2.11,0.78	2.50,2.49	2.50,2.49
Tool life TL_R, TL_F mins	-,175.70	10.18,5.01	93.35,10.26	175.65,175.65	176.65,174.48
Total Cost	148.73	244.28	174.23	148.72	148.69

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-8 (b) Optimal Cost and Process Variables for MS1 to MS5 for A=5 mm
$SF_F \leq 1$ μm, $SF_R \leq 6$ μm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	180.57	124.79	64.15	63.47
$Cost_F$ yens/piece	159.26	75.25	62.50	88.14	88.22
SF_F μm	1.00	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,2	4,1	2,1	1,1	1,1
Speed V_R, V_F m/min	-,268.82	171.28,290.88	216.00,325.76	216.00,250.52	216.00,250.52
Feed f_R, f_F mm/rev	-,0.213	0.780,0.301	0.354,0.279	0.341,0.202	0.346,0.203
Depth of cut d_R, d_F mm	-,2.5	1.178,0.288	2.14,0.72	2.19,2.80	2.17,2.83
Tool life TL_R, TL_F mins	-,114.82	10.18,4.360	98.80,9.16	109.4,182.72	105.09,184.57
Total Cost	159.26	255.82	187.29	152.29	151.69

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-9 (a) Optimal Cost and Process Variables for MS1 to MS5 for A=6 mm
$SF_F \leq 2\ \mu m, SF_R \leq 6\ \mu m$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	180.57	156.68	115.52	116.36
$Cost_F$ yens/piece	182.96	54.96	51.95	61.98	62.23
SF_F µm	1.99	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,3	4,1	2,1	2,1	2,1
Speed V_R, V_F m/min	-,219.88	171.28,235.31	217.70,254.20	208.92,217.86	208.00,217.00
Feed f_R, f_F mm/rev	-,0.358	0.780,0.394	0.266,0.439	0.404,0.354	0.402,0.353
Depth of cut d_R, d_F mm	-,2.0	1.178,1.288	2.61,0.78	1.938,2.123	1.94,2.12
Tool life TL_R, TL_F mins	-,83.15	10.18,31.64	200.0,10.26	75.08,94.69	77.50,95.70
Total Cost	182.96	235.53	208.63	177.50	178.59

SF: Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-9 (b) Optimal Cost and Process Variables for MS1 to MS5 for A=6 mm
$SF_F \leq 1$ µm, $SF_R \leq 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	180.57	159.47	106.64	106.74
$Cost_F$ yens/piece	192.75	67.04	62.50	80.12	80.46
SF_F µm	0.84	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,2	4,1	2,1	2,1	2,1
Speed V_R, V_F m/min	-,259.55	171.28,297.75	221.84,325.76	199.35,267.95	199.50,267.01
Feed f_R, f_F mm/rev	-,0.176	0.780,0.246	0.256,0.279	0.471,0.212	0.471,0.212
Depth of cut d_R, d_F mm	-,3.0	1.178,1.28	2.64,0.72	1.72,2.55	1.72,2.55
Tool life TL_R, TL_F mins	-,200.0	10.18,30.29	200.0,9.16	53.00,119.68	53.15,121.38
Total Cost	192.75	247.61	221.97	186.76	187.20

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-10(a) Optimal Cost and Process Variables for MS1 to MS5 for A=7 mm
$SF_F \leq 2$ µm, $SF_R \leq 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	225.72	181.98	137.38	137.42
$Cost_F$ yens/piece	206.07	53.54	51.95	68.69	68.66
SF_F µm	1.77	2.00	2.00	1.80	1.80
Passes n_R, n_F	-,3	5,1	3,1	2,1	2,1
Speed V_R, V_F m/min	-,216.00	171.28,240.77	214.47,254.20	216.00,216.00	216.00,216.00
Feed f_R, f_F mm/rev	-,0.310	0.780,0.407	0.370,0.439	0.314,0.314	0.314,0.314
Depth of cut d_R, d_F mm	-,2.33	1.178,1.11	2.07,0.78	2.33,2.33	2.33,2.33
Tool life TL_R, TL_F mins	-,136.82	10.18,22.71	88.93,10.26	137.50,137.50	137.59,137.24
Total Cost	206.07	279.26	233.93	206.07	206.08

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-10(b) Optimal Cost and Process Variables for MS1 to MS5 for A=7 mm
$SF_F \leq 1$ μm, $SF_R \leq 6$ μm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	225.72	183.25	125.18	124.80
$Cost_F$ yens/piece	234.37	65.16	62.50	84.97	85.06
SF_F μm	1.00	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,3	5,1	3,1	2,1	2,1
Speed V_R, V_F m/min	-,271.75	171.28,304.66	215.29,325.76	216.00,256.98	216.00,256.90
Feed f_R, f_F mm/rev	-,0.217	0.780,0.255	0.365,0.279	0.352,0.206	0.354,0.206
Depth of cut d_R, d_F mm	-,2.33	1.178,1.11	2.09,0.72	2.15,2.70	2.14,2.72
Tool life TL_R, TL_F mins	-,99.80	10.28,22.38	91.03,9.16	100.02,155.81	98.80,156.96
Total Cost	234.37	290.88	245.75	210.15	209.86

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-11(a) Optimal Cost and Process Variables for MS1 to MS5 for A=8 mm
$SF_F \leq 2$ μm, $SF_R \leq 6$ μm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	270.86	213.51	174.60	174.42
$Cost_F$ yens/piece	242.94	52.39	51.95	61.97	62.18
$S F_F$ μm	1.34	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,3	6,1	3,1	3,1	3,1
Speed V_R, V_F m/min	-,225.35	171.28,247.34	216.00,254.20	209.80,217.86	209.80,217.81
Feed f_R, f_F mm/rev	-,0.247	0.780,0.423	0.301,0.439	0.398,0.353	0.398,0.352
Depth of cut d_R, d_F mm	-,2.67	1.178,0.932	2.406,0.78	1.958,2.123	1.95,2.14
Tool life TL_R, TL_F mins	-,199.9	10.18,15.34	153.6,10.26	77.24,94.69	76.65,96.48
Total Cost	242.94	323.25	265.46	236.57	236.60

SF: Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-11(b) Optimal Cost and Process Variables for MS1 to MS5 for A=8 mm
$SF_F \leq 1$ µm, $SF_R \leq 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	270.86	215.59	165.64	166.19
$Cost_F$ yens/piece	251.12	63.52	62.50	80.12	80.52
SF_F µm	1.00	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,3	6,1	3,1	3,1	3,1
Speed V_R, V_F m/min	-,259.55	171.28,312.98	216.00,325.76	203.54,267.95	203.54,267.95
Feed f_R, f_F mm/rev	-,0.208	0.780,0.264	0.279,0.279	0.440,0.212	0.436,0.212
Depth of cut d_R, d_F mm	-,2.67	1.178,0.932	2.43,0.72	1.815,2.55	1.82,2.52
Tool life TL_R, TL_F mins	-,145.7	10.18,15.65	159.1,9.16	62.33,119.69	62.63,119.89
Total Cost	251.12	334.38	278.09	245.76	246.71

SF: Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-12(a) Optimal Cost and Process Variables for MS1 to MS5 for A=10 mm
$SF_F \le 2$ μm, $SF_R \le 6$ μm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	361.15	271.05	233.68	235.76
$Cost_F$ yens/piece	297.46	53.64	51.95	61.97	62.11
SF_F μm	1.60	1.99	2.00	2.00	2.00
Passes n_R, n_F	-,4	8,1	4,1	4,1	4,1
Speed V_R, V_F m/min	-,216.00	171.28,266.38	216.00,254.20	210.24,217.00	209.80,217.00
Feed f_R, f_F mm/rev	-,0.285	0.780,0.469	0.319,0.439	0.395,0.354	0.391,0.354
Depth of cut d_R, d_F mm	-,2.50	1.178,0.576	2.305,0.78	1.969,2.123	1.98,2.10
Tool life TL_R, TL_F mins	-,175.7	10.18,5.109	131.5,10.26	78.95,94.69	81.50,92.90
Total Cost	297.46	414.79	323.0	295.65	297.87

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-12(b) Optimal Cost and Process Variables for MS1 to MS5 for A=10 mm
$SF_F \le 1\ \mu m, SF_R \le 6\ \mu m$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	361.15	273.04	210.72	212.61
$Cost_F$ yens/piece	318.52	63.43	62.50	89.98	89.78
SF_F μm	1.00	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,4	8,1	4,1	3,1	3,1
Speed V_R, V_F m/min	-,268.82	171.28,337.08	216.00,325.76	216.00,246.97	216.00,246.97
Feed f_R, f_F mm/rev	-,0.213	0.780,0.293	0.316,0.279	0.306,0.200	0.302,0.201
Depth of cut d_R, d_F mm	-,2.50	1.178,0.576	2.32,0.72	2.38,2.86	2.39,2.80
Tool life TL_R, TL_F mins	-,114.8	10.18,5.770	134.6,79.16	147.6,200.00	151.8,194.96
Total Cost	318.52	424.58	335.54	300.70	302.39

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-13(a) Optimal Cost and Process Variables for MS1 to MS5 for A=15 mm
$SF_F \le 2\ \mu m, SF_R \le 6\ \mu m$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	541.73	418.28	374.64	374.41
$Cost_F$ yens/piece	437.60	52.11	51.95	62.67	62.95
$S\ F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,7	12,1	7,1	6,1	6,1
Speed V_R, V_F m/min	-,216.00	171.28,250.25	212.80,254.20	216.00,216.00	216.00,216.00
Feed f_R, f_F mm.rev	-,0.353	0.780,0.430	0.379,0.439	0.354,0.351	0.356,0.350
Depth of cut d_R, d_F mm	-,2.14	1.178,0.864	2.03,0.78	2.14,2.14	2.14,2.14
Tool life TL_R, TL_F mins	-,99.35	10.18,12.92	84.72,10.26	99.14,100.47	98.77,102.02
Total Cost	437.60	593.84	470.23	437.31	437.36

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-13(b) Optimal Cost and Process Variables for MS1 to MS5 for A=15 mm
$SF_F \leq 1$ µm, $SF_R \leq 6$ µm

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	541.73	419.47	363.90	364.03
$Cost_F$ yens/piece	477.78	63.03	62.50	80.12	80.11
SF_F µm	1.00	1.00	1.00	1.00	1.00
Passes n_R, n_F	-,6	12,1	7,1	6,1	6,1
Speed V_R, V_F m/min	-,268.82	171.28,316.66	213.30,325.76	214.66,267.96	214.66,267.96
Feed f_R, f_F mm/rev	-,0.213	0.780,0.268	0.377,0.279	0.369,0.212	0.369,0.212
Depth of cut d_R, d_F mm	-,2.50	1.178,0.864	2.04,0.72	2.07,2.55	2.07,2.54
Tool life TL_R, TL_F mins	-,114.8	10.18,13.39	85.77,9.16	89.42,119.69	89.43,119.48
Total Cost	477.78	604.76	481.97	444.02	444.14

SF:Surface finish, Subscripts: R=roughing, F=finishing

while imposing the final surface finish constraint of $SF_F \leq 2\mu$meters and $SF_F \leq 1\mu$meter. Parts (a) of Tables 4-6 to 4-13 show the results when $SF_F \leq 2\mu$meters and Parts (b) of each Table show the results when $SF_F \leq 1\mu$meter. Each Table contains the optimal roughing and finishing costs, their number of passes, speed, feed, depth of cut and tool life besides the finishing pass centerline average and the total cost for each strategy. Tables 4-14 and 4-15 present a summary of the optimal production costs for MS1 to MS5 for A=2,4,5,6,7,8,10,15,16,17,18,19,20 and 25 mm considering $SF_F \leq 2\mu$meters and $SF_F \leq 1\mu$meter respectively.

The production costs listed in Tables 4-14 and 4-15 are shown plotted against the respective values of 'A', in Fig. 4-5 and 4-6 for $SF_F \leq 2\mu$meters and $SF_F \leq 1\mu$meter respectively. These Figures show that for the two final surface finish restrictions, production costs by MS1, MS3, MS4 and MS5 increase for an increasing value of 'A'. For MS2, the production cost also increases with increasing 'A' except for a dip at A=6 mm, where the production cost is less than the cost for A=5 mm. A similar dip occurs at A=20 mm where the production cost is less than the cost for A=19 mm. The Figures also show that for any given value of 'A' and for $SF_F \leq 2\mu$meters or $SF_F \leq 1\mu$meter,

TABLE 4-14 Summary of Optimal Production Costs at various Values of 'A' for the Proposed Strategies, MS1 to MS5
$SF_F \leq 2$ µm, $SF_R \leq 6$ µm

A mm	MS1	MS2	MS3 Yens/piece	MS4	MS5
2	60.98	97.13	97.20	60.98	60.96
4	121.97	191.81	154.08	118.46	118.50
5	148.73	244.28	174.23	148.72	148.69
6	182.96	235.53	208.63	177.50	178.59
7	206.07	279.26	233.93	206.07	206.08
8	242.94	323.25	265.46	236.57	236.60
10	297.46	414.79	323.00	295.65	297.87
15	437.60	593.84	470.23	437.31	437.36
16	469.85	639.09	496.38	469.86	469.53
17	496.24	687.26	529.01	495.36	495.64
18	527.59	738.39	554.30	527.59	527.26
19	556.98	796.35	586.77	553.35	553.82
20	585.37	776.16	612.23	585.38	585.06
25	733.54	1012.77	760.13	730.44	730.97

TABLE 4-15 Summary of Optimal Production Costs at various Values of 'A' for the Proposed Strategies, MS1 to MS5
$SF_F \leq 1$ µm, $SF_R \leq 6$ µm

A mm	MS1	MS2	MS3 Yens/piece	MS4	MS5
2	74.71	107.92	108.22	74.71	74.72
4	145.13	201.75	165.80	128.19	128.22
5	159.26	255.82	187.29	152.29	151.69
6	192.75	247.61	221.97	186.76	187.20
7	234.37	290.88	245.75	210.15	209.86
8	251.12	234.38	278.09	245.76	246.71
10	318.52	424.58	335.54	300.70	302.39
15	477.78	604.76	481.97	444.02	444.14
16	502.23	649.41	508.87	473.90	478.79
17	534.31	696.95	541.05	503.15	503.03
18	564.94	749.39	566.77	531.77	531.40
19	595.55	810.14	599.30	562.23	562.34
20	628.79	787.89	624.69	589.67	589.20
25	785.32	1024.53	772.61	739.45	739.35

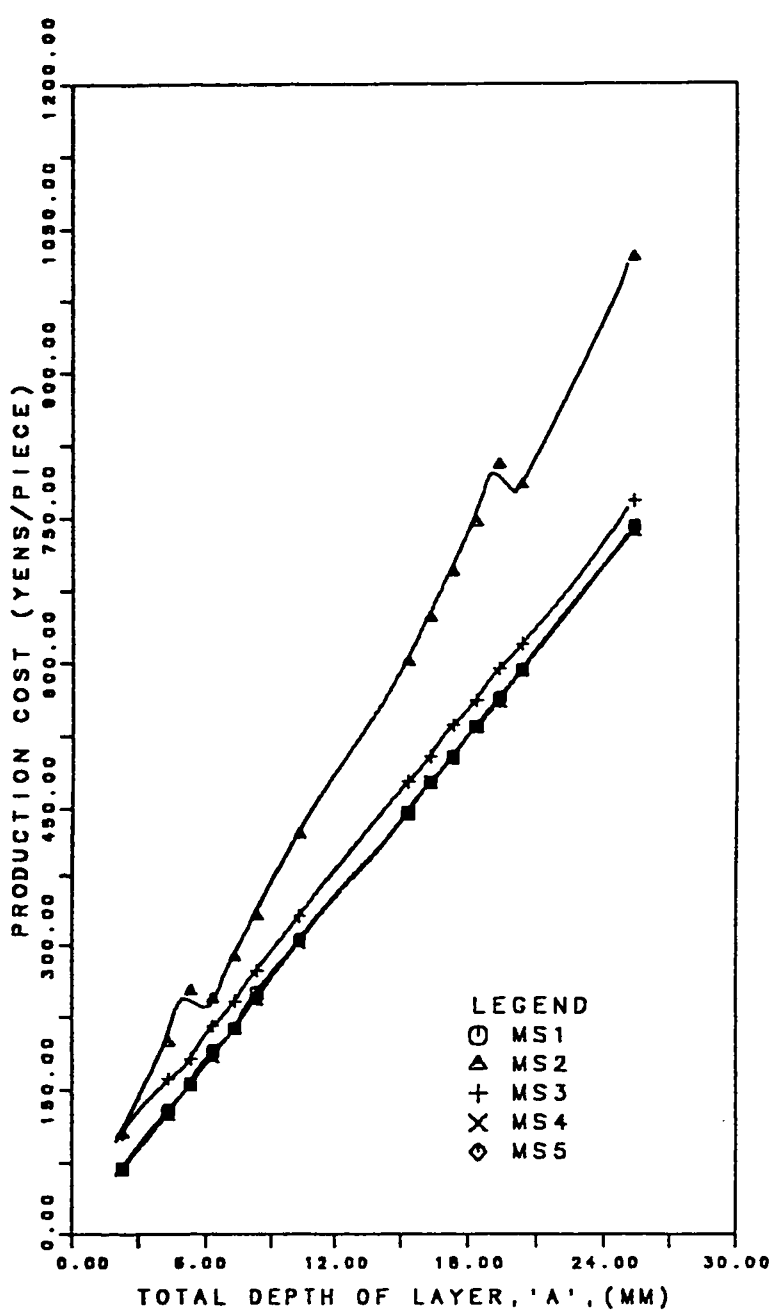

Figure 4-5 Variation of production cost with the total depth of layer to be removed for MS1 to MS5 and surface finish constraints,

$SF_R \leq 6\mu m, SF_F \leq 2\mu m$

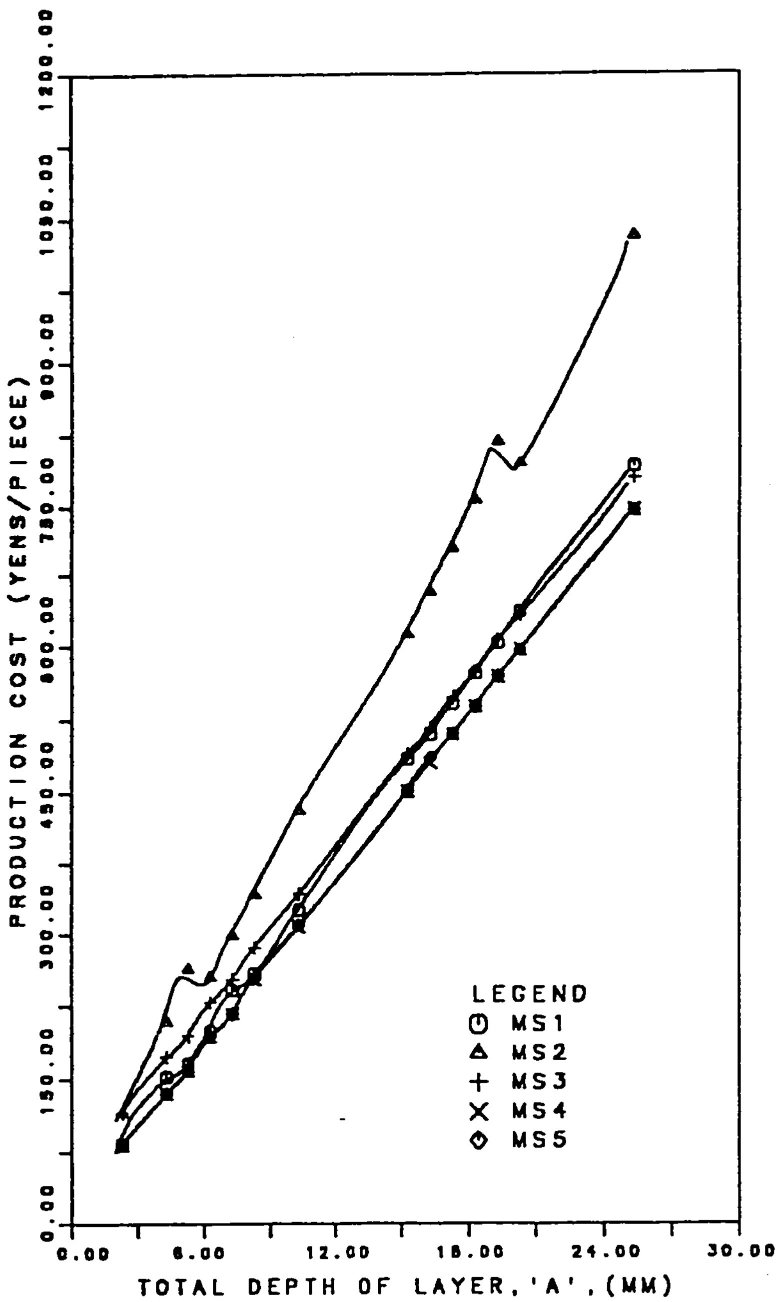

Figure 4-6 Variation of production cost with the total depth of layer to be removed for MS1 to MS5 and surface finish constraints,

$SF_R \leq 6\mu m, SF_F \leq 1\mu m$

MS2 provides the highest production costs followed by MS3 ,MS1 and MS4. MS4 and MS5 always incur equal and lowest production costs.

Fig. 4-5 shows that for $SF_F \leq 2\mu$**meters** MS1, MS4 and MS5 give approximately equal costs for the model considered. For example from Fig. 4-5 and Table 4-14 for A=4 mm, costs given by MS1, MS4 and MS5 are 121.97, 118.46 and 118.50 yens/piece respectively. Fig. 4-6 shows that for $SF_F \leq 1\mu$**meters** the cost for MS1 lies between the costs for MS3 and MS4 or is approximately equal to the costs for either MS3 or MS4. For example from Fig. 4-6 and Table 4-15, for A=10 mm the cost for MS1 is 5% less than the cost for MS3 but 6% higher than the cost for MS4. For A=18 mm from Table 4-15, the cost for MS1 is 564.94 yens/piece as compared to 566.77 yens/piece for MS3, while MS4 and MS5 provide costs of 531.77 and 531.40 yens/piece respectively. Although for any given 'A' and a final surface finish restriction the costs for MS4 and MS5 are equal for all practical purposes, the slight variations appearing in Tables 4-14 and 4-15 are attributed to the computational round-off errors while optimizing with MS5.

From the comparison of Table 4-14 with Table 4-15 it can be seen that for any given 'A', and any given strategy, the production cost when $SF_F \leq 1\mu$**meter**, is always greater than the production cost when $SF_F \leq 2\mu$**meters**. These results confirm to the practical machining situations where high surface finish requirements are accompanied by higher costs. Fig. 4-7 and 4-8 are bar charts which indicate the production costs under two values of final pass surface finish ($SF_F \leq 2\mu$**meters** and $SF_F \leq 1\mu$**meter**) for representative values of A=2 mm and 4 mm, and A=10 mm and 15 mm respectively. These bar charts clearly show that the lower production cost for $SF_F \leq 2\mu$**meters** as compared to the cost when $SF_F \leq 1\mu$meter, for MS1 to MS5.

4.5.3: Sensitivity of the Strategies to Tool Life Restriction

In order to determine the sensitivity of the proposed strategies to the tool life constraint, the general model was solved for MS1 to MS5 while considering two ranges of tool life constraints, when machining the total depth of A=2,4,5,6,7,8,10,15,16,17,18,19,20 and 25 mm.

For Range 1, tool life constraints for roughing and finishing are:

$$0 \leq TL_R \leq 25 \qquad (4.43)$$

$$0 \leq TL_F \leq 25 \qquad (4.44)$$

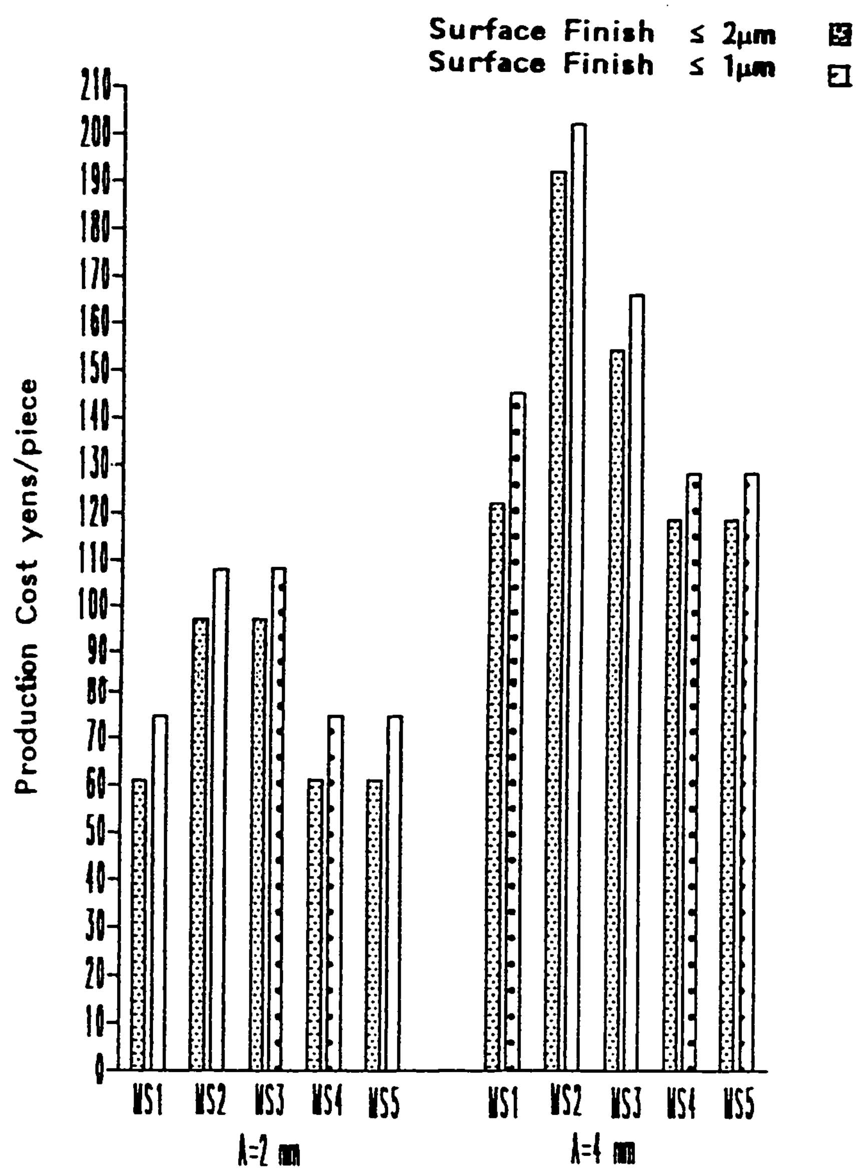

Figure 4-7 Production cost for two different values of final pass surface finish for MS1 to MS5, A=2 and A=4 mm.

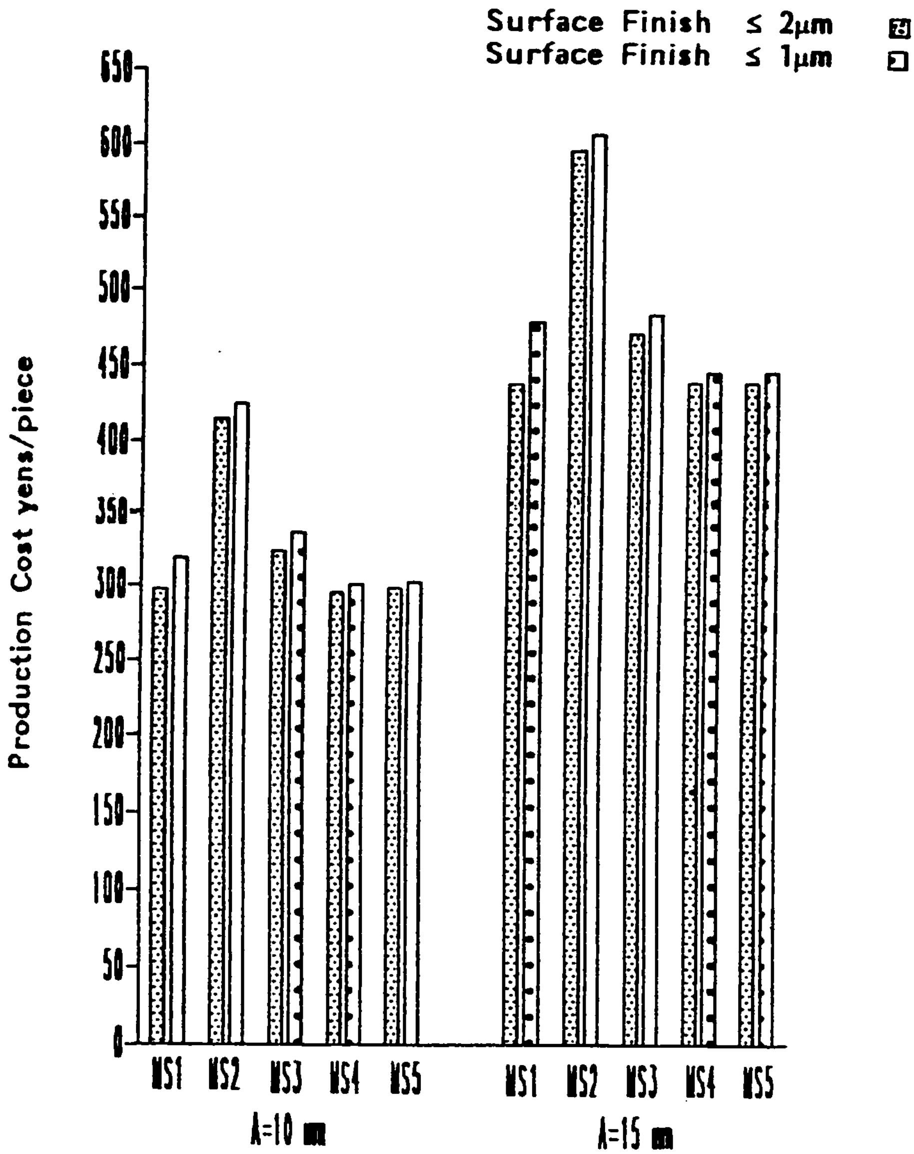

Figure 4-8 Production cost for two different values of final pass surface finish for MS1 to MS5, A=10 and A=15 mm.

and For Range 2, the tool life constraint for roughing and finishing are:

$$25 \leq TL_R \leq 45 \tag{4.45}$$

$$25 \leq TL_F \leq 45. \tag{4.46}$$

The objective functions and depth distribution constraints for each cutting strategy (MS1 to MS5) are given by Equations (4.12) to (4.27). The other constraints common to all strategies are given by Equations (4.28) to (4.35) and Equations (4.37) to (4.40). Surface finish is restricted to 6 μmeters in the roughing pass and 2 μmeters in the finishing pass.

Tables 4-16 to 4-23 list the detailed results of optimization of the strategies for A= 2,3,4,5,6,7,8,10 and 15 mm. Part (a) of Tables 4-16 to 4-23 show the results for Range 1 of the tool life and Part (b) of each Table show the results for Range 2 of the tool life. Each Table contains the optimal roughing and finishing costs, their number of passes, speed, feed, depth of cut and tool life, the finishing pass surface finish and the total cost for strategies MS1 to MS5. Tables 4-24 and 4-25 give the optimal production cost for MS1 to MS5 for A=2,4,5,6,7,8,10,15,16,17,18,19,20 and 25 mm. The production costs from the Tables 4-24 and 4-25 are plotted against the respective 'A',

TABLE 4-16(a) Optimal Cost and Process Variables for MS1 to MS5 for A=2 mm
$0 \le TL_R \le 25,\ 0 \le TL_F \le 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	45.145	45.25	45.15	45.15
$Cost_F$ yens/piece	105.56	51.99	51.95	51.97	51.98
SF_F µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,2	1,1	1,1	1,1	1,1
Speed V_R, V_F m/min	-,244.67	171.29,252.18	173.66,254.20	171.87,252.69	171.87,252.69
Feed f_R, f_F mm/rev	-,0.42	0.780,0.435	0.745,0.439	0.771,0.436	0.771,0.436
Depth of cut d_R, d_F mm	-,1.0	1.178,0.822	1.22,0.78	1.188,0.811	1.188,0.811
Tool life TL_R, TL_F mins	-,17.97	10.18,11.54	12.31,10.26	10.68,11.20	10.67,11.20
Total Cost	105.56	97.135	97.20	97.12	97.13

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-16(b) Optimal Cost and Process Variables for MS1 to MS5 for A=2 mm
$25 \leq TL_R \leq 45$, $25 \leq TL_F \leq 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	47.50	53.42	51.81	51.80
$Cost_F$ yens/piece	111.64	63.35	53.91	55.28	55.29
SF_F μm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,2	1,1	1,1	1,1	1,1
Speed V_R, V_F m/min	-,229.32	184.10,197.20	147.84,239.18	153.52,232.01	153.52,232.01
Feed f_R, f_F mm/rev	-,0.403	0.614,0.403	0.661,0.404	0.660,0.403	0.660,0.403
Depth of cut d_R, d_F mm	-,1.00	1.41,0.59	0.84,1.16	0.96,1.04	0.96,1.04
Tool life TL_R, TL_F mins	-,25.00	25.00,25.00	25.00,25.00	25.00,25.00	25.00,25.00
Total Cost	111.64	110.85	107.33	107.09	107.09

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-17(a) Optimal Cost and Process Variables for MS1 to MS5 for A=4 mm
$0 \leq TL_R \leq 25$, $0 \leq TL_F \leq 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	135.435	136.60	136.28	136.28
$Cost_F$ yens/piece	211.12	56.38	51.95	52.10	52.10
S F_F μm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,4	3,1	3,1	3,1	3,1
Speed V_R, V_F m/min	-,244.67	171.29,251.57	176.05,254.28	174.20,257.98	174.20,257.97
Feed f_R, f_F mm/rev	-,0.42	0.780,0.470	0.775,0.439	0.780,0.449	0.783,0.449
Depth of cut d_R, d_F mm	-,1.0	1.178,0.466	1.07,0.78	1.09,0.709	1.09,0.709
Tool life TL_R, TL_F mins	-,17.97	10.18,5.020	8.47,10.26	8.75,8.25	8.64,8.25
Total Cost	211.12	191.815	188.55	188.38	188.38

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-17(b) Optimal Cost and Process Variables for MS1 to MS5 for A=4 mm
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	95.00	95.25	95.00	95.00
$Cost_F$ yens/piece	166.36	54.08	53.91	54.04	54.05
SF_F µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,3	2,1	2,1	2,1	2,1
Speed V_R, V_F m/min	-,234.15	184.10,238.50	184.50,239.18	184.10,238.64	184.10,238.64
Feed f_R, f_F mm/rev	-,0.391	0.614,0.402	0.609,0.404	0.614,0.402	0.614,0.402
Depth of cut d_R, d_F mm	-,1.33	1.41,1.18	1.42,1.16	1.41,1.175	1.41,1.18
Tool life TL_R, TL_F mins	-,33.97	25.00,26.00	25.60,25.00	25.00,25.00	25.00,25.82
Total Cost	166.36	149.08	149.16	149.04	149.05

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-18(a) Optimal Cost and Process Variables for MS1 to MS5 for A=5 mm
$0 \le TL_R \le 25,\ 0 \le TL_F \le 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	180.58	142.06	137.17	137.13
$Cost_F$ yens/piece	263.90	63.72	51.95	53.91	53.92
S F_F µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,5	4,1	3,1	3,1	3,1
Speed V_R, V_F m/min	-,244.67	171.29,217.53	183.99,254.20	177.05,239.18	177.05,239.18
Feed f_R, f_F mm/rev	-,0.42	0.780,0.469	0.615,0.439	0.699,0.404	0.699,0.404
Depth of cut d_R, d_F mm	-,1.00	1.178,0.288	1.41,0.78	1.280,1.158	1.278,1.155
Tool life TL_R, TL_F mins	-,17.97	10.18,5.020	24.83,10.26	15.83,24.95	15.83,24.95
Total Cost	263.90	244.30	194.01	191.08	191.05

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-18(b) Optimal Cost and Process Variables for MS1 to MS5 for A=5 mm
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	142.50	145.42	145.41	145.40
$Cost_F$ yens/piece	218.59	59.40	53.91	53.91	53.94
SF_F µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,4	3,1	3,1	3,1	3,1
Speed V_R, V_F m/min	-,236.40	184.10,212.80	166.80,239.18	166.86,239.18	166.86,239.90
Feed f_R, f_F mm/rev	-,0.397	0.614,0.400	0.660,0.404	0.660,0.403	0.660,0.403
Depth of cut d_R, d_F mm	-,1.25	1.41,0.77	1.28,1.16	1.280,1.158	1.281,1.156
Tool life TL_R, TL_F mins	-,29.60	25.00,25.00	25.00,25.00	25.00,25.00	25.00,25.00
Total Cost	218.59	201.90	199.33	199.32	199.34

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-19(a) Optimal Cost and Process Variables for MS1 to MS5 for A=6 mm
$0 \le TL_R \le 25,\ 0 \le TL_F \le 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	225.73	183.50	181.46	181.44
$Cost_F$ yens/piece	316.68	80.31	51.95	53.08	53.08
$S\ F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,6	5,1	4,1	4,1	4,1
Speed V_R, V_F m/min	-,244.67	171.29,166.58	178.60,254.20	174.70,242.97	174.70,242.97
Feed f_R, f_F mm/rev	-,0.42	0.780,0.468	0.670,0.439	0.730,0.413	0.730,0.413
Depth of cut d_R, d_F mm	-,1.00	1.178,0.11	1.305,0.78	1.238,1.046	1.238,1.046
Tool life TL_R, TL_F mins	-,17.97	10.18,5.01	17.70,10.26	13.39,19.86	13.39,19.86
Total Cost	316.68	306.04	235.45	234.54	234.52

SF: Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-19(b) Optimal Cost and Process Variables for MS1 to MS5 for A=6 mm
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	190.00	153.40	146.82	146.72
$Cost_F$ yens/piece	227.07	71.51	53.91	56.85	56.86
$S F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,4	4,1	3,1	3,1	3,1
Speed V_R, V_F m/min	-,229.85	184.10,171.23	194.14,239.18	188.41,229.61	188.41,229.60
Feed f_R, f_F mm/rev	-,0.382	0.614,0.402	0.515,0.404	0.568,0.381	0.569,0.381
Depth of cut d_R, d_F mm	-,1.50	1.411,0.360	1.613,1.16	1.496,1.51	1.49,1.50
Tool life TL_R, TL_F mins	-,44.31	25.00,25.00	42.27,25.00	31.84,45.00	31.65,44.97
Total Cost	227.07	261.51	207.31	203.67	203.58

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-20(a) Optimal Cost and Process Variables for MS1 to MS5 for A=7 mm
$0 \le TL_R \le 25$, $0 \le TL_F \le 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	225.73	227.02	226.19	226.18
$Cost_F$ yens/piece	369.46	53.54	51.95	52.32	52.32
$S\ F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,7	5,1	5,1	5,1	5,1
Speed V_R, V_F m/min	-,244.67	171.29,240.77	175.01,254.20	173.47,247.98	173.47,247.98
Feed f_R, f_F mm/rev	-,0.42	0.780,0.407	0.726,0.439	0.748,0.424	0.748,0.424
Depth of cut d_R, d_F mm	-,1.00	1.178,1.110	1.24,0.78	1.216,0.916	1.215,0.916
Tool life TL_R, TL_F mins	-,17.97	10.18,22.72	13.65,10.26	12.13,14.83	12.13,14.83
Total Cost	369.46	279.27	278.97	278.51	278.50

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-20(b) Optimal Cost and Process Variables for MS1 to MS5 for A=7 mm
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	190.00	193.18	190.00	190.01
$Cost_F$ yens/piece	279.55	55.57	53.91	55.49	55.48
$S\ F_F$ μm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,5	4,1	4,1	4,1	4,1
Speed V_R, V_F m/min	-,232.30	184.10,233.34	186.56,239.18	184.10,233.57	184.00,234.00
Feed f_R, f_F mm/rev	-,0.387	0.614,0.389	0.587,0.404	0.614,0.390	0.614,0.391
Depth of cut d_R, d_F mm	-,1.40	1.41,1.360	1.460,1.16	1.412,1.35	1.412,1.34
Tool life TL_R, TL_F mins	-,38.05	25.00,35.69	28.79,25.00	25.00,35.19	25.00,34.94
Total Cost	279.55	245.57	247.09	245.49	245.49

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-21(a) Optimal Cost and Process Variables for MS1 to MS5 for A=8 mm
$0 \le TL_R \le 25$, $0 \le TL_F \le 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	270.87	271.68	234.14	234.14
$Cost_F$ yens/piece	376.53	52.39	51.95	53.91	53.91
$S F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,7	6,1	6,1	5,1	5,1
Speed V_R, V_F m/min	-,239.69	171.29,247.34	172.70,254.20	181.79,239.18	181.79,239.18
Feed f_R, f_F mm/rev	-,0.40	0.780,0.423	0.759,0.439	0.640,0.404	0.640,0.404
Depth of cut d_R, d_F mm	-,1.14	1.178,0.932	1.203,0.78	1.368,1.158	1.368,1.158
Tool life TL_R, TL_F mins	-,24.24	10.18,15.34	11.43,10.26	21.78,24.95	21.67,24.88
Total Cost	376.53	323.26	323.63	288.05	288.05

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-21(b) Optimal Cost and Process Variables for MS1 to MS5 for A=8 mm
$25 \le TL_R \le 45,\ 25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	237.50	238.85	205.20	205.20
$Cost_F$ yens/piece	332.73	56.50	53.91	56.85	56.84
S F_F μm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,6	5,1	5,1	4,1	4,1
Speed V_R, V_F m/min	-,234.15	184.10,225.98	176.05,239.18	194.59,229.61	194.59,229.60
Feed f_R, f_F mm/rev	-,0.391	0.614,0.403	0.637,0.404	0.511,0.381	0.511,0.381
Depth of cut d_R, d_F mm	-,1.33	1.41,0.95	1.368,1.16	1.622,1.510	1.621,1.504
Tool life TL_R, TL_F mins	-,33.97	25.00,25.00	25.00,25.00	43.16,45.00	43.10,44.72
Total Cost	332.73	294.00	292.76	262.05	262.04

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-22(a) Optimal Cost and Process Variables for MS1 to MS5 for A=10 mm
$0 \leq TL_R \leq 25$, $0 \leq TL_F \leq 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	361.16	322.97	318.92	318.93
$Cost_F$ yens/piece	481.95	53.64	51.95	53.91	53.90
$S\,F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,9	8,1	7,1	7,1	7,1
Speed V_R, V_F m/min	-,240.73	171.29,266.39	179.05,254.20	176.08,239.19	176.08,239.19
Feed f_R, f_F mm/rev	-,0.41	0.780,0.469	0.673,0.439	0.712,0.404	0.712,0.404
Depth of cut d_R, d_F mm	-,1.111	1.178,0.576	1.317,0.78	1.263,1.158	1.263,1.158
Tool life TL_R, TL_F mins	-,22.76	10.18,5.110	18.17,10.26	14.76,24.92	14.76,24.92
Total Cost	481.95	414.80	374.92	372.83	372.83

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-22(b) Optimal Cost and Process Variables for MS1 to MS5 for A=10 mm
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	332.50	291.98	285.24	285.30
$Cost_F$ yens/piece	392.78	92.50	53.91	56.85	56.83
SF_F μm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,7	7,1	6,1	6,1	6,1
Speed V_R, V_F m/min	-,231.60	184.19,127.97	187.23,239.18	184.24,229.60	184.24,229.60
Feed f_R, f_F mm/rev	-,0.385	0.614,0.401	0.580,0.404	0.612,0.381	0.612,0.381
Depth of cut d_R, d_F mm	-,1.428	1.411,0.130	1.473,1.16	1.414,1.510	1.415,1.499
Tool life TL_R, TL_F mins	-,39.88	25.00,25.00	29.86,25.00	25.00,45.00	25.24,44.54
Total Cost	392.78	425.00	345.89	342.09	342.13

SF:Surface finish, Subscripts: R=roughing,F=finishing

TABLE 4-23(a) Optimal Cost and Process Variables for MS1 to MS5 for A=15 mm
$0 \leq TL_R \leq 25$, $0 \leq TL_F \leq 25$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	541.74	504.40	470.64	470.64
$Cost_F$ yens/piece	700.39	52.11	51.95	53.91	53.91
S F_F µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,13	12,1	11,1	10,1	10,1
Speed V_R, V_F m/min	-,239.33	171.29,250.25	177.72,254.20	182.64,239.19	182.64,239.19
Feed f_R, f_F mm/rev	-,0.40	0.780,0.430	0.690,0.439	0.630,0.404	0.630,0.404
Depth of cut d_R, d_F mm	-,1.15	1.178,0.864	1.293,0.78	1.384,1.158	1.384,1.158
Tool life TL_R, TL_F mins	-,24.76	10.18,12.92	16.59,10.26	22.97,24.94	22.85,24.89
Total Cost	700.39	593.85	556.35	524.55	524.50

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-23(b) Optimal Cost and Process Variables for MS1 to MS5 for A=15 mm
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

	MS1	MS2	MS3	MS4	MS5
$Cost_R$ yens/piece	--	475.00	447.32	440.82	441.37
$Cost_F$ yens/piece	567.69	57.23	53.91	56.85	56.79
$S\ F_F$ µm	2.00	2.00	2.00	2.00	2.00
Passes n_R, n_F	-,10	10,1	9,1	9,1	9,1
Speed V_R, V_F m/min	-,229.85	184.10,222.52	190.44,239.18	188.53,229.60	188.53,229.60
Feed f_R, f_F mm/rev	-,0.382	0.614,0.403	0.550,0.404	0.567,0.381	0.566,0.381
Depth of cut d_R, d_F mm	-,1.50	1.41,0.90	1.537,1.16	1.498,1.510	1.501,1.48
Tool life TL_R, TL_F mins	-,44.31	25.00,25.00	35.35,25.00	32.00,44.00	32.22,43.71
Total Cost	567.69	532.23	501.23	497.67	498.16

SF:Surface finish, Subscripts: R=roughing, F=finishing

TABLE 4-24 Summary of Optimal Production Costs at various Values of 'A' for the Proposed Strategies, MS1 to MS5
$0 \le TL_R \le 25$, $0 \le TL_F \le 25$

A mm	MS1	MS2	MS3 Yens/piece	MS4	MS5
2	105.56	97.14	97.20	97.12	97.13
4	211.13	191.82	188.55	188.38	188.38
5	263.90	244.30	194.19	191.08	191.05
6	316.68	306.03	235.92	234.54	234.53
7	369.47	279.27	278.97	278.51	278.50
8	376.54	323.26	323.06	288.05	288.05
10	481.95	444.80	374.92	372.83	372.83
15	700.39	593.85	556.35	524.55	524.97
16	753.07	639.11	569.57	566.00	566.00
17	805.77	687.28	611.05	608.00	608.00
18	858.47	738.42	652.96	622.26	622.20
19	911.12	796.38	667.27	663.48	663.43
20	963.90	776.18	708.49	705.07	705.80
25	1182.31	1012.79	889.09	858.80	859.00

TABLE 4-25 Summary of Optimal Production Costs at various Values of 'A' for the Proposed Strategies, MS1 to MS5
$25 \le TL_R \le 45$, $25 \le TL_F \le 45$

A mm	MS1	MS2	MS3 Yens/piece	MS4	MS5
2	111.64	110.85	107.33	107.09	107.09
4	166.36	149.08	149.16	149.04	149.05
5	218.59	201.90	199.33	199.32	199.34
6	227.07	261.51	207.31	203.67	203.58
7	279.55	245.57	247.09	245.49	245.49
8	332.73	294.00	292.76	262.05	262.04
10	392.78	425.00	345.89	342.09	342.13
15	567.69	532.23	501.23	497.67	498.16
16	620.16	589.53	541.06	516.44	516.46
17	672.64	658.18	559.55	555.86	555.83
18	681.23	625.10	599.12	595.68	595.68
19	733.70	679.52	618.16	614.32	614.33
20	786.18	744.82	657.47	653.83	654.15
25	960.78	863.37	813.90	810.17	811.24

in Fig. 4-9 and 4-10 for tool life Ranges 1 and 2 respectively.

Figures 4-9 and 4-10 show that for each tool life range, the production cost for MS1, MS3, MS4 and MS5 increases with increasing values of 'A'. The only exception is MS2, where although the cost increases with increasing 'A', the production cost for A=7 mm is less than the cost for A=6 mm and the production cost for A=20 mm is less than the cost for A=19 mm for tool life Range 1. The cost of MS2 for A=7 mm is less than the cost for A=6 mm and the cost for A=18 mm is less than the cost for A=17 mm for tool life of Range 2.

Figures 4-9 and 4-10 also show that for the two tool life Ranges, the four strategies, MS1 to MS4 give production costs which vary with a given 'A'. However for any 'A' and a given tool life range, the costs given by MS1, MS2 and MS3 are either higher than or equal to the cost by MS4. The costs given by MS4 and MS5 for a given 'A' and a tool life range are always equal. The small difference in costs for MS4 and MS5 appearing in Tables 4-24 and 4-25 is attributeable to computational round off errors while solving with MS5.

The production costs listed in Tables 4-24 and 4-25 show that for a given strategy, the difference in the production costs between tool life Ranges 1 and 2 varies with the value of 'A'. Fig. 4-11 and 4-12 are the bar charts for the production cost for MS1 to MS5 for the

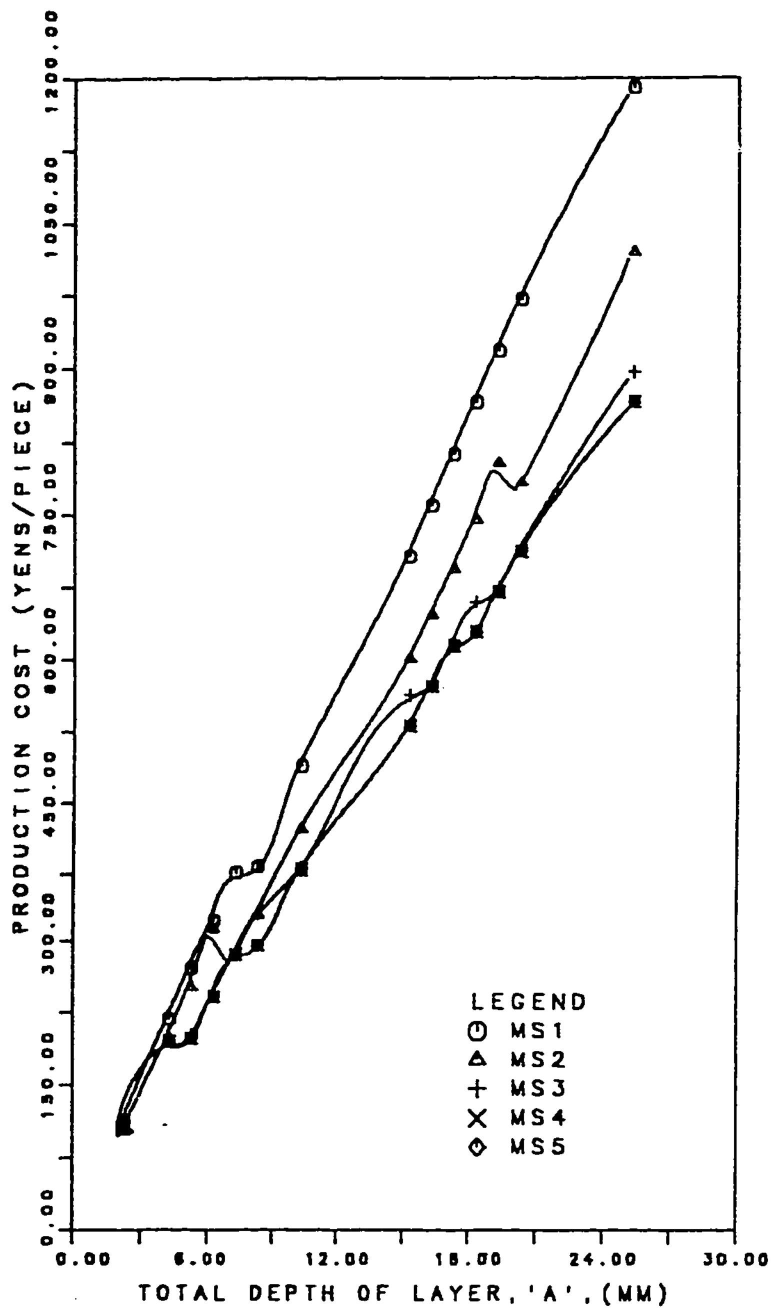

Figure 4-9 Variation of production cost with the total depth of layer to be removed for MS1 to MS5 and tool life constraints,

$$0 \le TL_R \le 25, 0 \le TL_F \le 25$$

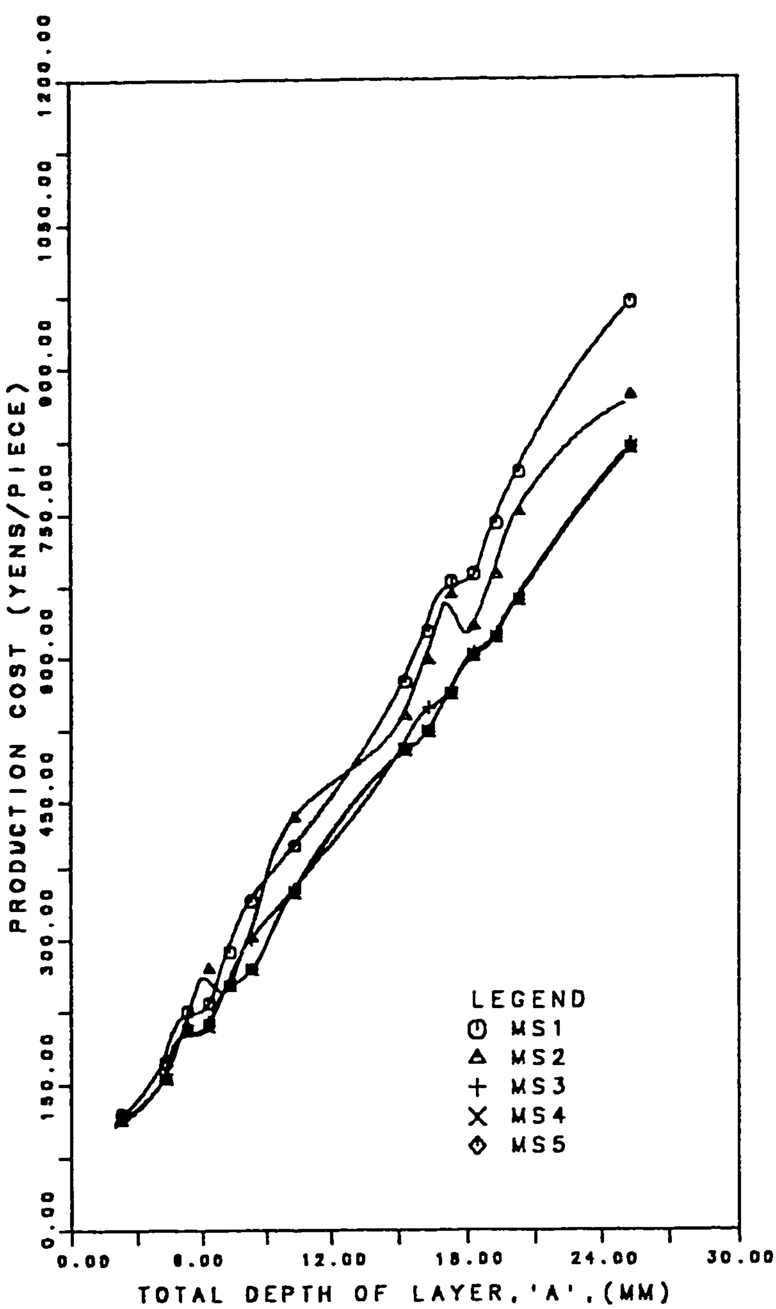

Figure 4-10 Variation of production cost with the total depth of layer to be removed for MS1 to MS5 and tool life constraints,

$$25 \leq TL_R \leq 45, 25 \leq TL_F \leq 45$$

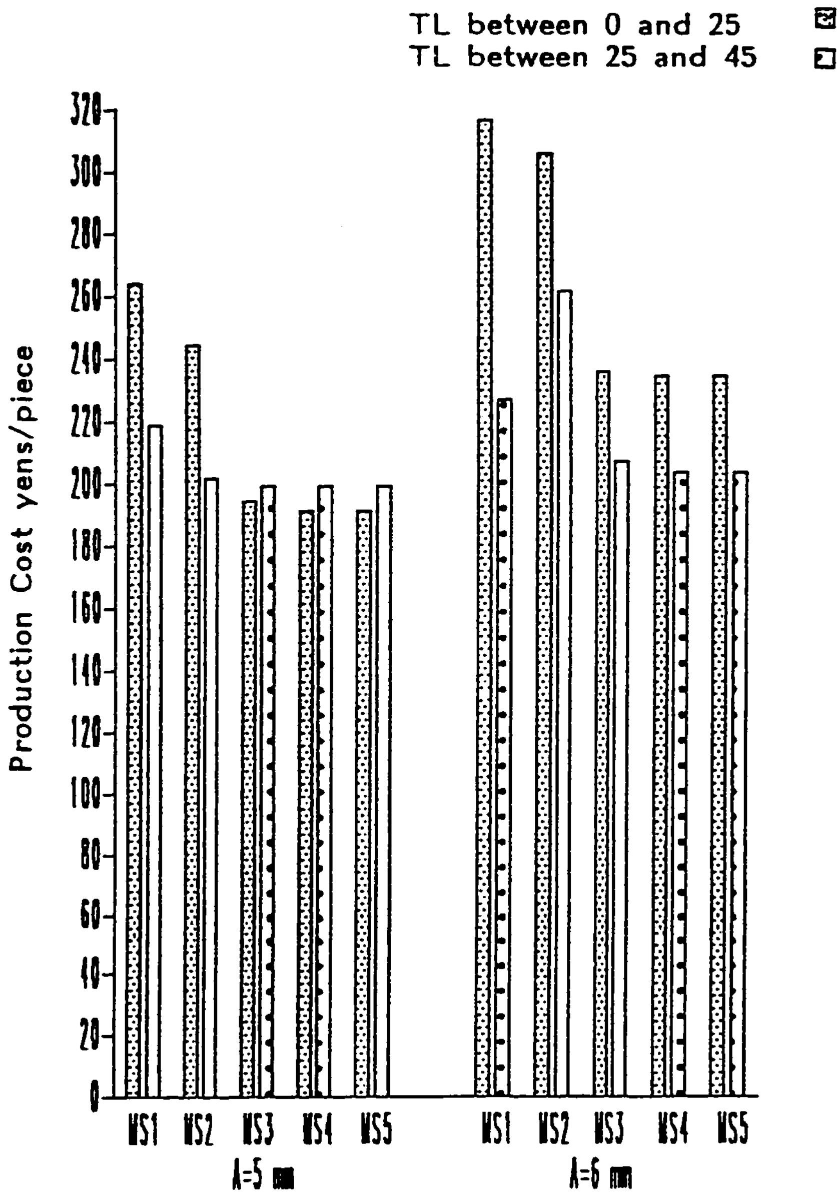

Figure 4-11 Production cost for two different restrictions on tool life for MS1 to MS5, A=5 and 6 mm.

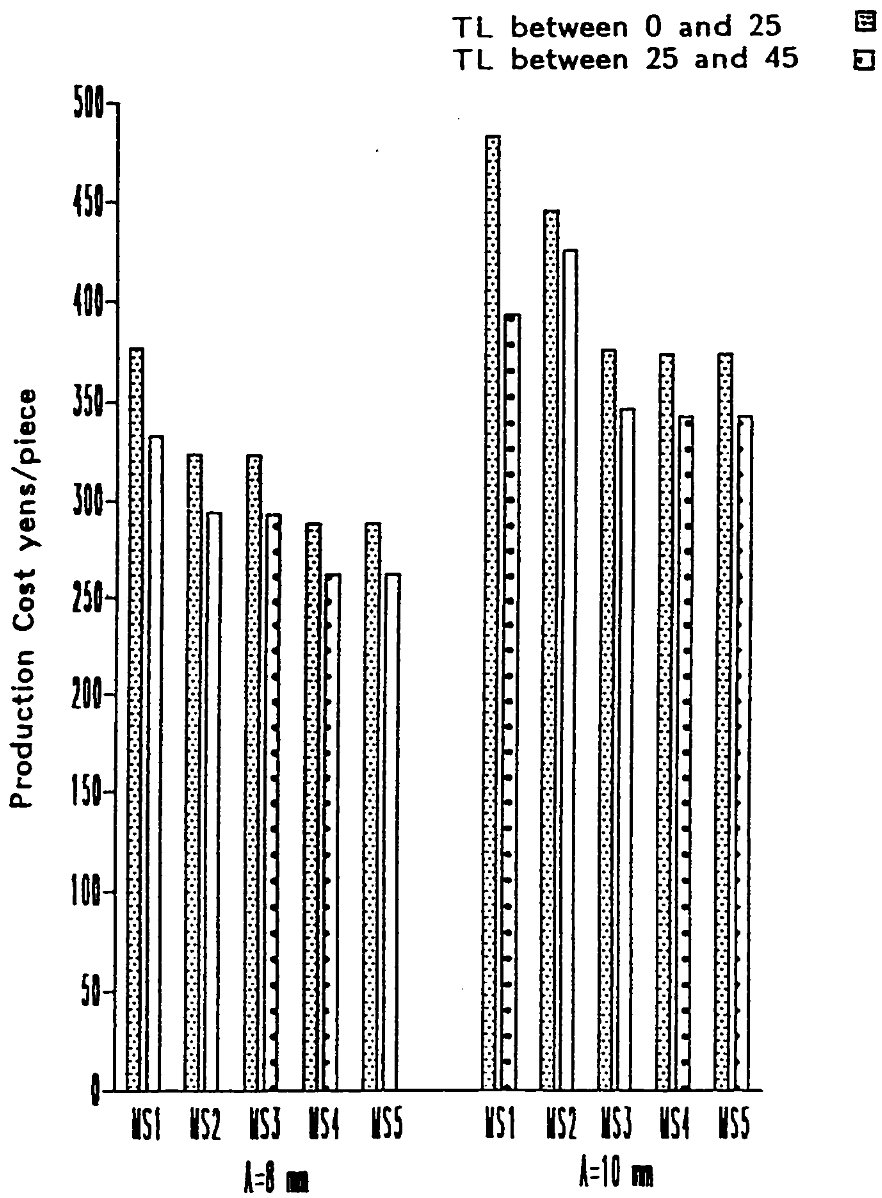

Figure 4-12 Production cost for two different restrictions on tool life for MS1 to MS5, A=8 and 10 mm.

two tool life Ranges, for representative values of A=5 mm and 6 mm and A=8 mm and 10 mm respectively. For a given value of 'A', these bar charts clearly show the variation in cost differences between tool life Range 1 and 2 provided by a given strategy.

4.5.4: Sensitivity to the Total Layer of Material to be Removed,'A'

Sections 4.5.2 and 4.5.3 covered the sensitivity of MS1 to MS5, to the final pass surface finish and the tool life restrictions. Fig. 4-5, 4-6, 4-9 and 4-10 show that the production cost increases with increasing 'A' for MS1, MS3, MS4 and MS5. However for MS2 the production cost also increases with increasing 'A' up to a certain value of 'A' where the production cost drops and then increases again with 'A' as shown as a dip in Fig. 4-5, 4-6, 4-9 and 4-10. The occurance of the dip can be attributed to the model considered and warrants further investigation to explain what appears to be related to the 'more for less paradox', i.e. removing more material for less cost.

4.6 DISCUSSION OF RESULTS AND CONCLUSION

Section 4.5 dealt with the sensitivity of the proposed multipass turning cutting strategies to final pass surface finish, tool life and total depth to be removed. Tables 4-6 to 4-15 and Fig. 4-5 and 4-6 show that for each of the two levels of final pass surface finish ($SF_F \le 2\mu$meters and $SF_F \le 1\mu$meter) the cost for MS1 is either higher than or equal to the cost for MS4, while the costs for MS2 and MS3 are always higher than the cost for MS4. Tables 4-16 to 4-25 and Fig. 4-9 and 4-10 show that for each of the two ranges of tool life for roughing and finishing passes ($0 \le TL \le 25$ and $25 \le TL \le 45$) the cost for MS1, MS2 and MS3 is either equal to or higher than the cost for MS4. For a given restriction on final pass surface finish or on the tool life the cost for MS4 is always equal to the cost for MS5. The slight variations between costs of MS4 and MS5 appearing in Tables 4-6 to 4-25 are attributed to computational round off errors while optimizing with MS5. Thus it is concluded that MS4 and MS5 are the most economical cutting strategies for the general multipass turning optimization model.

The equality of cost for MS4 and MS5 in Tables 4-6 to 4-25 present empirical evidence of the equivalence of MS4 and MS5. This equality of cost for MS4 and MS5 can be explained as follows: The depth distribution constraint for MS4 is given by:

$$n\, d_R + d_F = A \tag{4.47}$$

where n = 0,1,2,3..., is the number of roughing passes,

d_R = depth of the roughing pass and

d_F = depth of the finishing pass.

Subscripts R and F stand for roughing and finishing respectively.

For MS5 the depth distribution constraint is given by:

$$\sum_{i=1}^{m} d_i = A \tag{4.48}$$

where A = depth of layer to be removed and

m = the number of passes required to remove 'A'.

Equation (4.48) can be expanded to

$$\sum_{i=1}^{n} d_{R_i} + d_F = A \tag{4.49}$$

where n= m-1, so that we have m-1 roughing passes followed by a finishing pass. Depth of cut $d_{R_i} = d$, means that we have identical roughing passes, $\sum_{i=1}^{n} d_{R_i} = n\, d$, and Equation (4.48) reduces to (4.47). It has been shown that for any given value of 'A', under similar set of constraints and the depth distribution constraints of A=nd and $A = \sum_{i=1}^{n} d_i$, identical results are obtained for a given model. For the General model (Section 4.5.1), the Iwata's model (Section 3.2.1) and Hati and Rao's model (Section 3.2.2) the identical results for A=nd and $A = \sum_{i=1}^{n} d_i$ under similar set of constraint for each model, are shown in Table 4-26 to 4-28 respectively for selected value of 'A'. Identical results under similar constraints along with the depth of cut constraints A=nd and $A = \sum_{i=1}^{n} d_i$ are also obtained for Ermer's model (Section 3.2.4) and Ermer and Kromodihardjo's model (Section 3.2.5). The results of Tables 4-26 to 4-28 furnish an empirical evidence for the equivalence of Equation (4.47) and (4.48) and along with the equal results of production cost for MS4 and MS5 in Tables 4-6 to 4-25 lead us to conjecture that MS4 is equivalent to MS5. However, this conclusion requires an analytical proof in a future work.

TABLE 4-26 Comparison of Results for the Constraints A=nd and $A = \sum_{i=1}^{n} d_i$ with the General Model, A=5 mm

$A = \sum_{i=1}^{n} d_i$					A=nd				
n	V m/min	f mm/rev	d mm	Cost yens/pc	n	V	f	d	Cost
2	216.00	0.285	2.49	148.70	2	216.0	0.285	2.50	148.70
	216.00	0.285	2.50			216.0	0.285	2.50	
3	196.7	0.493	1.66	156.53	3	196.7	0.493	1.66	156.53
	196.7	0.493	1.66			196.7	0.493	1.66	
	196.7	0.493	1.66			196.7	0.493	1.66	
4	175.4	0.721	1.25	181.81	4	175.4	0.721	1.25	181.81
	175.4	0.721	1.25			175.4	0.721	1.25	
	175.4	0.721	1.25			175.4	0.721	1.25	
	175.4	0.721	1.25			175.4	0.721	1.25	

TABLE 4-27 Comparison of Results for the Constraints A=nd and $A = \sum_{i=1}^{n} d_i$ with Iwata's Model, A=4 mm

$A = \sum_{i=1}^{n} d_i$					A=nd				
n	V m/min	f mm/rev	d mm	Cost yens/pc	n	V	f	d	Cost
1	216.00	0.149	4.00	182.06	1	216.0	0.149	4.00	182.06
2	216.00	0.386	2.00	166.06	2	216.0	0.386	2.00	166.06
	216.00	0.386	2.00			216.0	0.386	2.00	
3	324.4	0.4105	1.33	189.00	3	324.4	0.4105	1.33	189.00
	324.4	0.4105	1.33			324.4	0.4105	1.33	
	324.4	0.4105	1.33			324.4	0.4105	1.33	
4	331.0	0.4105	1.00	245.63	4	331.0	0.4105	1.00	245.63
	331.0	0.4105	1.00			331.0	0.4105	1.00	
	331.0	0.4105	1.00			331.0	0.4105	1.00	
	331.0	0.4105	1.00			331.0	0.4105	1.00	

TABLE 4-28 Comparison of Results for the Constraints A=nd and $A = \sum_{i=1}^{n} d_i$ with Hati and Rao's model, A=5 mm

$A = \sum_{i=1}^{n} d_i$					A=nd				
n	V fpm	f ipr	d in	Cost dollars/pc	n	V	f	d	Cost
2	146.31	0.375	2.50	151.57	2	146.3	0.375	2.50	151.57
	146.31	0.375	2.50			146.3	0.375	2.50	
3	132.5	0.592	1.667	160.42	3	132.5	0.592	1.667	160.42
	132.5	0.592	1.667			132.5	0.592	1.667	
	132.5	0.592	1.667			132.5	0.592	1.667	
4	127.4	0.750	1.25	176.41	4	127.4	0.750	1.25	176.41
	127.4	0.750	1.25			127.4	0.750	1.25	
	127.4	0.750	1.25			127.4	0.750	1.25	
	127.4	0.750	1.25			127.4	0.759	1.25	

Mathematically MS5 is the most general strategy because it permits the total depth 'A' to be removed in 'n' independent passes. However the cutting variables (V,f and d) and the model constraints associated with each pass multiply with the number of passes and increases model complexity for MS5. On the other hand MS4 consists of making 'n' identical optimal roughing passes and one final finishing pass. This reduces the model complexity and saves computational time. For the General model and the conditions of Tables 4-6 to 4-25 the approximate CPU (Central Processing Unit) time on the IBM 3033 mainframe computer at KFUPM for MS5 was found to be 5 to 10 times higher than for MS4. For example for the condition of Table 4-13(a) the approximate CPU time for MS5 was 250 secs as compared to 25 secs for MS4. Although MS5 is more general mathematically, computational savings with MS4 combined with the conjectured equivalence of MS5 to MS4 dictates the preference of MS4 over MS5.

5. GENERALIZED MACHINING OPTIMIZATION PACKAGE

5.1 INTRODUCTION

This chapter presents a computer program package which enables the choice of any of the machining strategies presented in chapter 4, for the optimization of multipass turning operations. The package uses a generalized model, which is an extension of the model in Reference [9], from which a specific problem can be derived by selecting the appropriate objective and relevant set of constraints during a multipass turning operation. The package is capable of using the objective functions of minimum production cost, minimum production time (or the maximum production rate) and maximum metal removal rate. If the user of the package encounters a model which is different from the proposed general model, then he has to modify some of the subroutines to customize the package to his requirements. Since the optimization package handles multipass turning operations, when each pass is taken at different set of constraints, dictated by the design or the end user's requirements, the package can also be employed for the optimization of multi-tool or multi-machine operations, where different passes with corresponding sets of constraints, are performed on different machines or by different tools. However, the objective functions for multi-operation and multi-tool operations have to be derived for a particular application.

As stated in chapter 3, multi-variable constrained optimization methods based on non-linear programming techniques are well suited to machining optimization models. Both SUMT and PC based version of the generalized reduced gradient (GRG), the general integrated optimizer (GINO) have been found to be efficient and reliable methods for machining optimization, amongst those taken up for study in chapter 3. Although GINO performs well in terms of the accuracy of the final solution and freedom from a starting vector, it does not accept a model with more than thirty statements. This becomes a limitation for multipass turning operations when each pass has to satisfy its own set of constraints, which may cause the problem to exceed thirty input statements. Due to the lack of the source program for GINO, it is not possible to extend it beyond thirty input statements. On the other hand, the availability of the SUMT source program allows a very large number of input statements and offers greater flexibility in terms of output and input format as compared to GINO. SUMT is a Fortran based program and can be used both on the mainframe and a PC with a Fortran compiler. Moreover the use of user supplied subroutines with SUMT, in contrast to GINO, also justifies its choice for the package. Therefore the proposed computer package uses SUMT as an optimization method.

Section 5.2 lists the general machining optimization model used in the package. Section 5.3 describes the package, its parameters, the subroutines required and presents program inputs and outputs for an example problem. Section 5.4 discusses the results and Section 5.5 presents the listing of the package .

5.2 THE GENERAL MACHINING OPTIMIZATION MODEL

The general machining optimization model for multipass turning operations uses minimum production cost or minimum production time or maximum material removal rate as objectives and combines all possible operational and end user's requirements through the constraint formulations. Each constraint is represented by an empirical equation. The coeffecients and exponents of these equations are determined experimentally and are dependent upon the machine-tool-workpiece system. For the general model the formulations are in accordance with those of Reference [9]. The general model gives a chance to the user to select a set of constraints suitable for his operational and design requirements.

The general multipass turning machining optimization model with the objectives of minimum production cost or minimum production time or maximum material removal rate together with the constraints of speed, feed, depth of cut, tool life, cutting force, power, stability, temperature and surface finish has the following formulations:

a) The Production Cost per pass is given by: [12]

$$\text{Cost} = \text{Machining Cost} + \text{Tooling Cost} + \text{Handling Cost} \quad (5.1)$$

$$\text{Cost} = C_o t_m + \left(\frac{t_m}{TL}\right)(C_o t_e + C_t) + C_o t_p \quad (5.2)$$

where C_o is the operating cost,

C_t is the tool cost per cutting edge

t_e is the tool changing time and

t_p is the handling time.

The machining time in turning is given by

$$t_m = \frac{\pi D L}{V f} \quad (5.3)$$

where 'L' and 'D' are the length and diameter of the workpiece respectively.

The tool life equation is expressed empirically by:

$$TL = C_1 \, V^{b_1} \, f^{b_2} \, e^{b_3 f} \, d^{b_4} \tag{5.4}$$

where C_1 , b_1 , b_2 , b_3 , and b_4 are empirical constants. Putting Equations (5.3) and (5.4) in Equation (5.2) the general form of the production cost for Turning operations is given by:

$$Cost = \alpha_o \, V^{-1} \, f^{-1} + \alpha_1 \, V^{\beta_1} \, f^{\beta_2} \, e^{\beta_3 f} \, d^{\beta_4} + C_p \tag{5.5}$$

where the α_i s and β_j s are constant values given by:

$$\alpha_o = C_o \, \pi \, D \, L \tag{5.6}$$

$$\alpha_1 = \left(\frac{\pi \, D \, L}{C_1}\right)(C_o t_e + C_t) \tag{5.7}$$

$$\beta_1 = -(b_1 + 1) \tag{5.8}$$

$$\beta_2 = -(b_2 + 1) \tag{5.9}$$

$$\beta_3 = -b_3 \tag{5.10}$$

$$\beta_4 = -b_4 \tag{5.11}$$

The tool life equation given by [9] has a factor involving the power of 'e', whereas tool life equations given by References [5,12,22] do not have a factor with the power of 'e'. Therefore for tool life equations which do not involve the power of 'e', b_3 is taken as zero in Equation (5.4) and consequently β_3 is also taken as zero in the objective of cost given by Equation (5.5).

b) The objective of production time is given by:

Time = Machining time + Tool Changing Time + Handling Time (5.12)

$$\text{Time} = t_m + \frac{t_m t_e}{TL} + t_p \qquad (5.13)$$

Substituting Equations (5.3) and (5.4) in Equation (5.13) the general form of the production time for turning operations is given by:

$$\text{Time} = \alpha_3 V^{-1} f^{-1} + \alpha_4 V^{\beta_5} f^{\beta_6} e^{\beta_7 f} d^{\beta_8} + t_p \qquad (5.14)$$

It can be seen that the expression of production cost given by Equation (5.5) and the expression of production time given by Equation (5.14) have the same form i.e each value of a constant in one equation corresponds to one value in the other equation. The exponent β_7 is taken as zero for the models in which the tool life equation does not have a power of 'e'. The α_i s and β_j s are constant values given by:

$$\alpha_3 = \pi \, D \, L, \tag{5.15}$$

$$\alpha_4 = \left(\frac{\pi \, D \, L}{C_1}\right)(t_e) \tag{5.16}$$

$$\beta_5 = -(\, b_1 + 1 \,) \tag{5.17}$$

$$\beta_6 = -(\, b_2 + 1 \,) \tag{5.18}$$

$$\beta_7 = -b_3 \tag{5.19}$$

$$\beta_8 = -b_4 \tag{5.20}$$

c) The objective of material removal rate (MRR) is given by:

$$MRR = 10^{-6} \, V \, f \, d \, \left(\frac{m^3}{min}\right) \tag{5.21}$$

where V, f and d are the cutting speed, feed and depth of cut respectively and d<<D.

d) The general formulations of the constraints are as follows:

i) The maximum and minimum cutting speeds

$$V_{min} \le V \le V_{max} \tag{5.22}$$

ii) The maximum and minimum feeds

$$f_{min} \le f \le f_{max} \tag{5.23}$$

iii) The maximum and minimum depth of cut

$$d_{min} \le d \le d_{max} \tag{5.24}$$

iv) The maximum cutting force

$$F_c \le F_{c_{max}} \tag{5.25}$$

where, $$F_c = C_2 \, V^{a_1} \, f^{a_2} \, d^{a_3} \tag{5.26}$$

v) The maximum power consumption

$$P_c \le P_{c_{max}} \tag{5.27}$$

where, $$P_c = C_3 \, F_c \, V \tag{5.28}$$

vi) The stable cutting region related to the cutting surface

$$C_4 V^{a_5} f^{a_6} \geq C \tag{5.29}$$

vii) Tool life constraint:

This is given by equation 5.4.

ix) Temperature constraint:

$$T \leq T_{max} \tag{5.30}$$

where,
$$T = C_5 V^{a_7} f^{a_8} d^{a_9} \tag{5.31}$$

x) Surface finish, centerline average constraint:

$$SF = C_6 V^{a_{10}} f^{a_{11}} d^{a_{12}} \tag{5.32}$$

xi) The depth distribution constraint:

Depth distribution constraints for the proposed cutting strategies have already been outlined in chapter 4.

5.3 DESCRIPTION OF THE MACHINING OPTIMIZATION PACKAGE

a) Purpose

This package optimizes the objectives of production cost or production time or material removal rate given by Equations (5.5), (5.14) and (5.21) subject to a set of the model constraints given by Equation (5.4) and Equations (5.22) to (5.32).

b) Procedure

The main program, MAIN, reads: i) options for the cutting strategy and the objective function ii) values of the operational parameters like the length and diameter of the workpiece, the operating cost etc. iii) values of the exponents and coefficients for the constraints and iv) calculates the values of the coeffecients and constraints for the objective function. The number of machining variables (V,f and d) and constraints, the constraint limits and all other options are read in sub-program MAIN1. The control is then passed to the optimization block, which consists of the method of SUMT [31,34] and incorporates three subroutines developed to handle the cutting strategies proposed in chapter 4. SUMT has already been described in section 3.1.1. After SUMT reaches the convergence limits, the control is transferred to the printing block in the package. A simplified flow diagram of the package is shown in Fig. 5-1. Table 5-1 indicates the relationship existing between the various subroutines for the SUMT optimization method.

c) *Subroutines required*

Subroutine MAIN1, called from the main program, coordinates SUMT with the following subroutines. These subroutines were taken from [31].

Sub-program BODY coordinates all sub-programs.

Sub-program CHCKER computes first derivatives of objective function.

Sub-program CONVRG checks for convergence.

Sub-program DIFF1 computes first derivatives by central difference.

Sub-program DIFF2 computes second derivatives by central difference.

Sub-program ESTIM estimates lagrangian multiplier values and final solution extrapolation.

Sub-program EVALU evaluates the objective function and constraints.

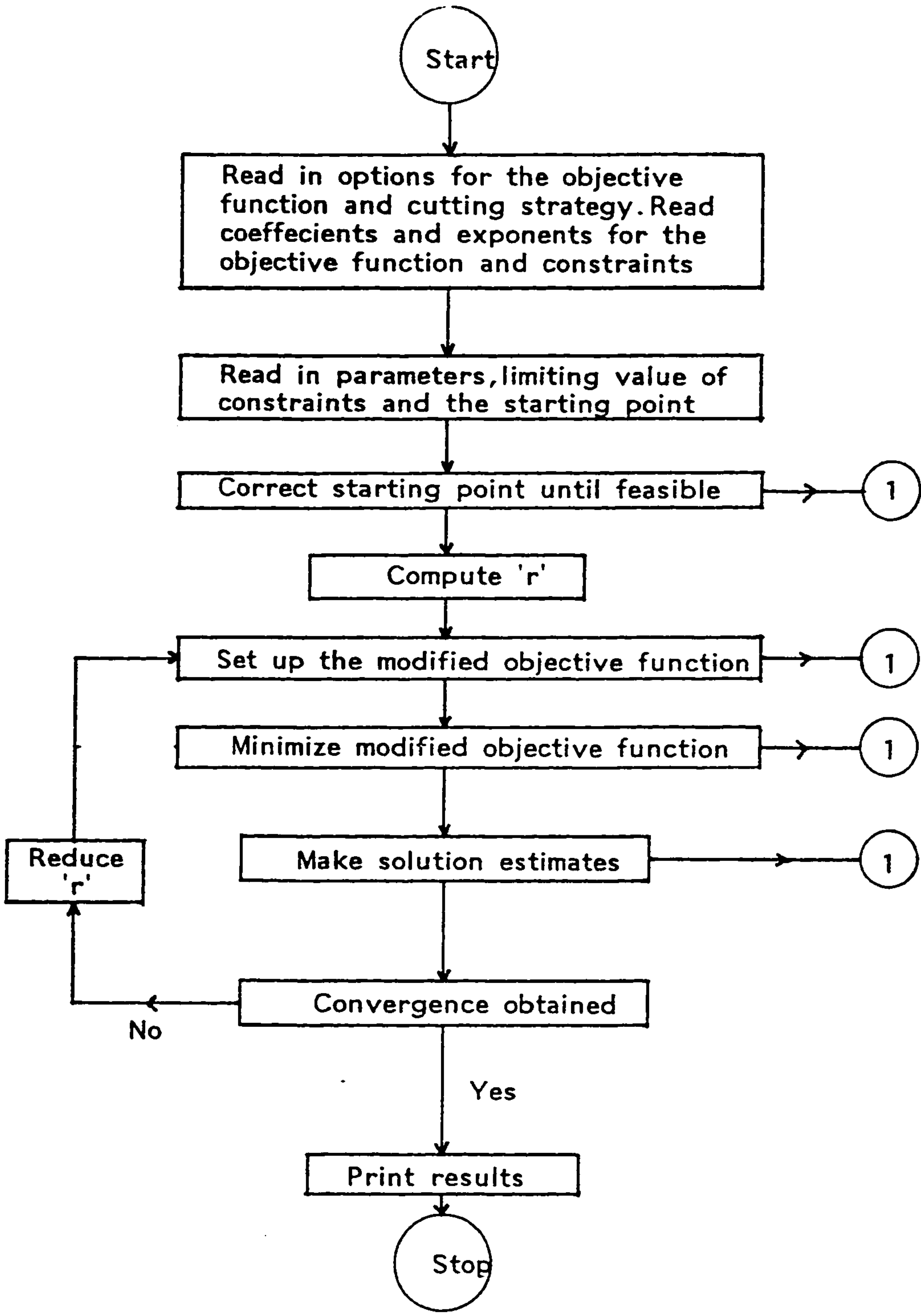

Figure 5-1: A simplified flow diagram of the machining optimization package

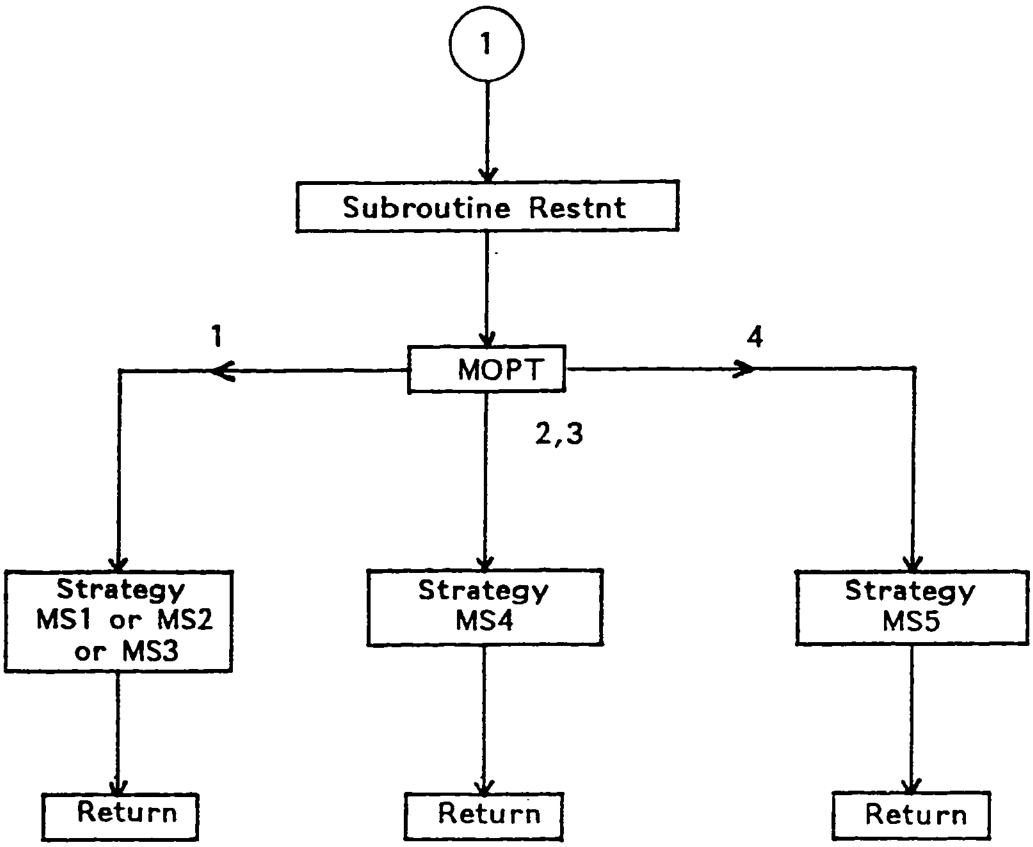

Figure 5-1 (contd.) A simplified flow diagram of the machining optimization package

TABLE 5-1 Relationship Existing Between the Subroutines for the SUMT Optimization Method [34]

Calls \ Called by	BODY	CHCKER	CONVRG	DIFF1	DIFF2	ESTIM	EVALU	FEAS	FINAL	GRAD1	GRAD	INVERS	MATRIX	OPT	OUTPUT	PEVALU	REJECT	RESTNT	RHOCOM	SECORD	STORE	XMOVE
MAIN	*	*					*	*							*						*	
BODY			*			*	*	*			*			*	*	*			*			*
CHCKER				*	*					*			*									
CONVRG																						
DIFF1										*								*				
DIFF2																						
ESTIM							*		*						*		*				*	
EVALU																		*				
FEAS	*						*								*			*				
FINAL																						
GRAD										*												
OPT							*								*		*					
PEVALU											*	*										
RHOCOM										*										*		
SECORD													*									
XMOVE												*								*	*	

Sub-program FEAS determines feasibility of the starting point; if starting point is not feasible, a feasible point is sought. If no feasible point is possible a request is made for a new input vector.

Sub-program FINAL checks for convergence.

Sub-program GRAD computes first derivatives of penalty function.

Sub-program INVERS solves linear system of equations.

Sub-program OPT performs one dimensional search by Golden section method.

Sub-program OUTPUT prints out results of each iteration.

Sub-program PEVALU computes value of penalty function and dual.

Sub-program REJECT returns stored values to original locations.

Sub-program RHOCOM computes initial value of 'r'(r is positive constant in Equation (3.4)).

Sub-program SECORD computes second derivatives of penalty function.

Sub-program STORE stores values of current point.

Sub-program XMOVE determines search directions.

Sub-program GRAD1 evaluates analytical first derivatives for objective function and constraints.

Sub-program MATRIX evaluates analytical second derivatives for objective function and constraints.

Sub-programs SET, STORE and TCHECK in the SUMT are replaced by the Sub-program SETIME in the package, which interacts with the clock in the IBM mainframe computer at KFUPM.

The following subroutines have been developed and added to SUMT for the developed package.

Sub-program RESTNT calls Sub-programs for different values of MOPT (option for strategies MS1 to MS5).

Subroutine RESNDS is called for strategies MS1, MS2 and MS3.

Subroutine RESAND is called for strategy MS4.

Subroutine RESUMD is called for strategy MS5.
Subroutines RESNDS, RESAND, RESUMD contain the objective functions and constraint formulations for the respective strategies.

Subroutine MAIN2 is called for strategy MS4, when it is desired to find the value of number of passes that result in the least of the optimal minimum values of the objective function.

Subroutines PRIND, PRIAND and PRISUM print the name of the strategy used, optimal value of the objective function, values of the corresponding machining variables and values of surface finish and tool life for each pass.

d) Description of Parameters

MOPT: Option key for the cutting strategy

= 1 for MS1, MS2 and MS3.

= 2 and 3 for MS4

= 4 for MS5

NOBJ: Option key for the objective function

= 1 for production cost

= 2 for production time

= 3 for material removal rate

NNP: Number of passes for a given value of total depth of layer, 'A'.

NND: Total number of values of NNP (number of times NNP is changed)

MJJ: Number of constraint values in the model

BOU: Vector for the upper and lower limits on the constraints

Values of constants, coefficients and exponents input to the program:

D, L, C_o, t_e, C_t, t_p	For production cost
D, L, t_e, t_p	For production time

As stated earlier in Section 5.2, the expressions of production cost and production time have the same form. Therefore the expression of production cost would hold good for time, if C_o is taken as unity and C_t as zero, and the values of constants and exponents for the cost are replaced by appropriate values for production time.

C_1, b_1, b_2, b_3, b_4	For Tool life
C_2, a_1, a_2, a_3, a_4	For Cutting force
C_3	For Power
C_4, a_5, a_6	For Stability
C_5, a_7, a_8, a_9	For Temperature
$C_6, a_{10}, a_{11}, a_{12}$	For surface finish

NI	Card reader unit number
NO	Printer unit number

EPSI Tolerance, used to decide if an unconstrained minimum has been achieved for each subproblem. Usually taken as 1E-9.

RHOIN Possible initial value of 'r' in Equation (3.4). Usually set at 1.

THETAO Tolerance, used to decide if the solution to the problem has been approximated, usually taken as 1E-6.

RATIO Parameter, used to compute consecutive values of 'r' .Usually set at 16.0

TMMAX Maximum amount of time for solving problem Not used in the package.

M Number of inequality constraints.

N Number of independent variables.

MZ Number of equality constraints

X Independent variables.

NT1 Option key for 'r' values. Usually set at 3 so that the value of 'r' is 1.

NT2 Option key for constraints:

= 1 The requirements that the trivial constraints are to be automatically included in the problem.

= 2 The only constraints on the problem are those input by the user.

NT3 Option key for printout:

= 1 Standard printout

= 2 For additional printout

(normally set to 1)

NT4 Option key on final convergence:

= 1 Final convergence is determined on the basis of current solution to the subproblem

= 2 Final convergence is determined on the basis of the first order estimates

= 3 Final convergence is determined on the basis of the second order estimates

(normally set to 1)

NT5 Option key on final convergence

(normally set to 1)

NT6 Option key for next problem

(normally set to 1)

NT7 Option key for extrapolation as follows

= 1 No extrapolation

= 2 Extrapolate through last two minima

= 3 Extrapolate through last three minima

(normally set to 1)

NT8 Not used

NT9 Option key for subproblem convergence

(normally set to 1)

NT10 Option key on problem linearity

= 1 At least one nonlinear constraint

= 2 Linear constraint

= 3 Linear constraints and linear objective function

XEP1 Finite difference parameter. 0.0001 is satisfactory

XEP2 Iteration improvement limit. Normally set to zero

NEXOP1 Key for checking derivatives as follows:

= 1 Solve problems without checking derivatives

= 2 Solve problem after checking only first derivatives

= 3 Do not solve problem after checking only first derivatives

= 4 Solve problem after checking first and second derivatives

= 5 Do not solve problem after checking first and second derivatives

NEXOP2 Key for choosing unconstrained minimization technique:

= 1 The method for minimizing unconstrained penalty function is to be the generalized Newton-Raphson method as modified to handle indefinite Hessian matrices. This method requires function values, first and second derivatives.

= 2 Same as 1, except that an orthogonal move is made because of an indefinite Hessian matrix is added to the orthogonal move vector.

= 3 Steepest descent is used to minimize the penalty function.

= 4 The method for minimizing the unconstrained penalty function is McCormick's modification of the Fletcher Powell method.

e) *Dimension and Format Requirements*

The package is currently dimensioned to handle an optimization problem with up to 100 variables and 200 constraints. All input to the package is in free format.

f) Output

The package prints; i)The name of the strategy used, ii) total depth of layer to be removed, iii) number of passes, iv) the value of the production cost or production time or metal removal rate v) the optimal values of the machining variables, and vi) optimal values of surface finish and tool life. The user can also print the options used, starting point information and intermediate result printouts, until convergence is achieved. CPU time is also printed at the end of each problem.

g) Summary of User Requirements

i) Determine the values of the coeffecients and exponents for the objective functions and the constraints and the values for the restrictions on the constraints in the vector BOU.

ii) Specify other parameters as listed in Section 'd' for the description of parameters.

iii) Adjust the dimension and common statements, if required by the problem.

h) *Test Problem*:

A test problem with the following values of exponents and coeffecients for the objective function of cost and the constraints of speed, feed, depth of cut, tool life, cutting force, temperature, stability, power, and surface finish was optimized for a total depth of layer of 4 mm. For the Test problem the constants for the objective function and the constraints are taken from [9,12,38]. The parameters and constants are as follows.

For the objective function of cost:

DIA = 100, LENGTH = 250, C_o = 50, t_e = 0.5, C_t = 75, C_p = 10

For the tool life:

C_1 = 4.02E11, b_1 = -3.887, b_2 = 0, b_3 = -5.884, b_4 = 1.117

For cutting force: C_2 = 290.73, a_1 = -0.1013, a_2 = 0.725, a_3 = 1

For power: C_3 = 0.059

For stability: C_4 = 1, a_5 = 2, a_6 = 1

For Temperature: C_5 = 132.00, a_7 = 0.4, a_8 = 0.2, a_9 = 0.105

For Surface finish:

$C_6 = 141.65, a_{10} = -0.587, a_{11} = 1.179, a_{12} = 0.165$

The other parameters and options to the test problem are as follows. The first column gives the value of the parameter or the option which are stated the in second column.

1,2 or 4	MOPT
1	NOBJ
1	NND
3	NNP
1E-9 1 1E-6 4.00 60.0	EPSI,RHOIN,THETAO,RATIO,TMAMX
28 6 0	M,N,MZ
1 1 1 1 2	NT1,NT2,NT3,NT4,NT5
2 1 1 1 1	NT6,NT7,NT8,NT9,NT10
0.0001 0	XEP1,XEP2
4 4	NEXOP1,NEXOP2
18	MJJ
1005.3	upper limit on speed
14.13	lower limit on speed
5.6	upper limit on feed
0.01	lower limit on feed
4.0	upper limit on depth of cut

0.0	lower limit on depth of cut
7.5	maximum limit on power
170	maximum limit on cutting force
1000	maximum temperature allowed
45.0	upper limit on tool life, roughing
0.0	lower limit on tool life, roughing
6.0	maximum surface finish , roughing
2.0	maximum surface finish, finishing
2230.5	stability limitation
4.0	total depth of layer to be removed
3.999	lower limit on depth distribution constraint
45.0	upper limit on tool life, finishing
0.0	lower limit on tool life, finishing
190.0 0.573 1.42	Values of the input variables
250.3 0.370 1.15	(initial variables)
	for speed, feed, and depth of
	cut for roughing and finishing
	passes respectively.

The outputs for this optimization problem for three strategies of MS1, MS4 and MS5 are contained in the following section. Each set of three 'X' values in the output represents values of speed, feed and depth of cut for each pass. The CPU time given at the end of each output represents approximate execution time in seconds for the IBM 3033 mainframe computer at KFUPM.

j) <u>Sample Program Outputs</u>

i) FOR STRATEGY NO.1

TOTAL DEPTH OF LAYER = 4.00 MM

NO OF PASSES = 3

DEPTH OF CUT FOR EACH PASS = 1.33 MM

TOTAL PRODUCTION COST = 1.66023543E+02 YENS/PIECE

FINAL X VALUES:

X(1)=2.34071872E+02 X(2)=3.91648110E-01

SURFACE FINISH = 2.00 MICROMETERS

TOOL LIFE = 10.86 MINS

CPU TIME 2.16

ii) FOR STRATEGY NO.4

VALUES FOR BOTH ROUGHING AND FINISHING PASSES

TOTAL DEPTH OF LAYER = 4.00 MM

NO OF ROUGHING PASSES = 2

THE COST FOR ROUGHING = 9.70705249E+01 YENS/PIECE

THE COST FOR FINISHING = 5.57753284E+01 YENS/PIECE

THE TOTAL PRODUCTION COST = 1.52845915E+02 YENS/PIECE

FINAL X VALUES:

X(1)=1.89999729E+02 X(2)=5.74287392E-01 X(3)=1.42339009E+00

X(4)=2.50289920E+02 X(5)=3.69606872E-01 X(6)=1.15321800E+00

SURFACE FINISH FOR ROUGHING PASSES = 3.59 MICROMETERS

SURFACE FINISH FOR THE FINISHING PASS = 1.75 MICROMETERS

TOOL LIFE FOR THE ROUGHING PASSES = 28.22 MINS

TOOL LIFE FOR FOR THE FINISHING PASS = 25.48 MINS

CPU TIME 19.89

iii) FOR STRATEGY NO.5

TOTAL DEPTH OF LAYER = 4.00 MM

NO OF DIVS OF DEPTH OF CUT = 3

TOTAL PRODUCTION COST = 1.53135221E+02 YENS/PIECE

FINAL X VALUES:

X(1)=1.89999733E+02 X(2)=5.80180078E-01 X(3)=1.39597945E+00

X(4)=1.89999733E+02 X(5)=5.80180078E-01 X(6)=1.39597945E+00

X(7)=2.50299901E+02 X(8)=3.60831015E-01 X(9)=1.20704626E+00

FOR PASS NO 1 SURFACE FINISH = 3.62 MICROMETERS

FOR PASS NO 1 TOOL LIFE = 26.68 MINS

FOR PASS NO 1 COST = 48.29 YENS/PIECE

FOR PASS NO 2 SURFACE FINISH = 3.62 MICROMETERS

FOR PASS NO 2 TOOL LIFE = 26.68 MINS

FOR PASS NO 2 COST = 48.29 YENS/PIECE

FOR PASS NO 3 SURFACE FINISH = 1.72 MICROMETERS

FOR PASS NO 3 TOOL LIFE = 28.23 MINS

FOR PASS NO 3 COST = 56.56 YENS/PIECE

CPU TIME   47.10

5.4 USING PERSONAL COMPUTER FOR THE MACHINING OPTIMIZATION PACKAGE

The test problem presented in Section 5.3, was also run with the optimization package on a MYCOM personal computer (PC) with 640K RAM (Random Access Memory) using Microsoft Fortran 4.0 compiler. The results for each strategy were the same as obtained by using the package on the IBM mainframe as listed in Section 5.3. For the test problem of Section 5.3, the execution times for the package on the PC for the three strategies MS1, MS4 and MS5 were 90 secs, 840 secs and 2220 secs respectively as compared to 2, 20 and 47 secs for MS1, MS4 and MS5 respectively for the IBM mainframe computer at KFUPM. Although the execution time for the package on the PC is high as compared to the execution time on the mainframe computer, the adaptation of the package to the PC makes it available for use in a laboratory or an industrial environment where the mainframe computer may not be available.

5.5 MACHINING OPTIMIZATION PACKAGE PROGRAM LISTING

```
C.******************************************************************
C.***********GENERALISED MACHINING OPTIMIZATION PACKAGE**************
C.******************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER*4 ICDE,TIMER
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,P0,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON /EXPOPT/ NEXOP1,NEXOP2,XEP1,XEP2
      COMMON /DEVC/NI,NO,NP
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/EXTM/M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14,M15
      COMMON/PARM/ALPHA0,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
      DIMENSION FF(20),XOPT(600)
      DATA ICDE/90000/
      CALL SETIME(ICDE)
      NI=5
      NO=6
      READ(NI,*) MOPT,NOBJ
C.    IF MOPT EQ 1 NDS
C.    IF MOPT EQ 2 ND1+D2
C.    IF MOPT EQ 3 ND1+D2 SEARCH
C.    IF MOPT EQ 4 SUMDS
C.    THE FOLLOWING VALUES ARE FOR THE OBJECTIVE FUNCTIONS
C.    AND CONSTRAINTS
      READ (NI,*) DIA,LNGTH,CO,TE,CT,CM
      READ (NI,*) C1,B1,B2,B3,B4
      READ (NI,*) C2,A1,A2,A3
      READ (NI,*) C3
      READ (NI,*) C4,A5,A6
      READ (NI,*) C5,A7,A8,A9
      READ (NI,*) C6,A10,A11,A12
      ALPHA0=(CO*3.14159*DIA*LNGTH)/(1000)
      ALPHA1=((3.14159*DIA*LNGTH)/(1000*C1))*(CO*TE+CT)
      BETA1=-(B1+1)
      BETA2=-(B2+1)
```

```
      BETA3=-B3
      BETA4=-B4
      IF(MOPT.EQ.3) GO TO 22
      CALL MAIN1
      GO TO 33
 22   CALL MAIN2
 33   CALL TSTIME(TIMER)
      RTIMER=(90000-TIMER)/100.0
      WRITE(9,1) RTIMER
  1   FORMAT(' ',/,10X,'***CPU TIME*** ',F15.2,/)
      STOP
      END
      SUBROUTINE MAIN1
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,P0,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON /EXPOPT/ NEXOP1,NEXOP2,XEP1,XEP2
      COMMON /DEVC/NI,NO,NP
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/EXTM/M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14,M15
      COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
      DIMENSION FF(20),XOPT(600)
      READ(NI,*) NND
      DO 1804 MJ=1,NND
      READ(NI,*) NNP
0104  READ(NI,*)EPSI,RHOIN,THETAO,RATIO,TMMAX
      READ(NI,*) M,N,MZ
      IF (N) 40,40,107
107   CONTINUE
      READ(NI,*)NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      READ(NI,*) XEP1,XEP2
      READ (NI,*) NEXOP1,NEXOP2
      READ(NI,*) MJJ
      DO 2834 MNJ=1,MJJ
      READ (NI,*) BOU(MNJ)
2834  CONTINUE
      READ(NI,*) (X(I),I=1,N)
      WRITE(NO,004)
```

```
004    FORMAT (1H1,10X,'NONLINEAR PROGRAMMING ROUTINE  --  SUMT
      1VERSION',/)
       WRITE(NO,005) N,M,MZ,TMMAX,RHOIN,RATIO,EPSI,THETAO
005    FORMAT (//,2X,10HPARAMETERS,//,2X,4HN = ,I2,4X,4HM = ,I2,4X,5HMZ
      1 = ,I2,4X,8HTMMAX = ,F8.3,/,2X,6HRHO = ,E10.4,4X,8HRATIO = ,E10.4,
      24X,9HEPSILON = ,E11.4,4X,8HTHETA = ,E10.4 )
       WRITE (NO,006) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10,NEXOP1,
      1NEXOP2
006    FORMAT (//,2X,16HOPTIONS SELECTED,//,2X,6HNT1 = ,I1,3X,6HNT2 = ,
      1I1,3X,6HNT3 = ,I1,3X,6HNT4 = ,I1,3X,6HNT5 = ,I1,/,2X,6HNT6 = ,
      2I1,3X,6HNT7 = ,I1,3X,6HNT8 = ,I1,3X,6HNT9 = ,I1,3X,7HNT10 = ,I1,
      3/,2X,9HNEXOP1 = ,I1,3X,9HNEXOP2 = ,I1 )
       WRITE (NO,007) XEP1,XEP2
007    FORMAT (//,2X,37HTOLERANCES FOR DIFFERENCING AND MOVES ,//,2X,
      17HXEP1 = ,E10.4,4X,7HXEP2 = ,E10.4 )
       IF (MOPT.EQ.4) CALL MODF
       NTCTR=0
       NP1=N+1
       NM1=N-1
       CALL TIMEC
       NPHASE=4
       CALL EVALU
       PO=0.0
       G=0.0
       H =0.0
       RSIGMA=0.0
       CALL OUTPUT (2)
       CALL STORE
       IF (NEXOP1.GT.1) CALL CHCKER
       IF (NEXOP1.EQ.3) GO TO 40
       IF (NEXOP1.EQ.5) GO TO 40
       CALL FEAS
       WRITE(7,0908) NPHASE
0908   FORMAT(/,I2,/)
       GO TO (30,30,30,30,1804) ,NPHASE
30     NPHASE=2
       NTCTR=0
       CALL BODY
       IF(MOPT.EQ.1)GO TO 2887
       IF(MOPT.EQ.4)GO TO 2888
       IF(MOPT.EQ.2)GO TO 2889
       CALL PRIEXM
       GO TO 1804
2887   CALL PRINDS
       GO TO 1804
2888   CALL PRISUM
       GO TO 1804
```

```
2889  CALL PRIAND
      GO TO 1804
1804  CONTINUE
 40   RETURN
      END
      SUBROUTINE BODY
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/ F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON /CONPAR/ NF1,NF2,NF3
      COMMON /DEVC/NI,NO,NP
      NF2=2
      NF3=2
      MN=0
      NUMINI=0
      CALL RHOCOM
      CALL EVALU
10    CALL XMOVE
      GO TO (30,20), NT3
20    CALL TIMEC
      CALL OUTPUT(1)
      GO TO 40
30    CALL TCHECK
40    GO TO(50,50,50,200),NSATIS
50    CALL CONVRG (N1)
      GO TO(60,10,125), N1
60    GO TO (70,80), NT3
70    CALL TIMEC
      CALL OUTPUT(1)
80    NUMINI=NUMINI+1
      MN=0
      GO TO (190,90,90),NPHASE
90    CALL ESTIM
      GO TO(100,110,120), NT4
100   CALL FINAL (NF1)
      GO TO (130,140), NF1
110   GO TO (130,140), NF2
120   GO TO (130,140), NF3
125   NPHASE=5
130   RETURN
140   RHO=RHO/RATIO
      CALL PEVALU
```

```
      IF (NUMINI-2) 10,150,150
150   GO TO (10,160,160),NT7
160   CALL GRAD(2)
      CALL OPT
      GO TO (180,170), NT3
170   WRITE (NO,210)
210   FORMAT (//,2X,'MOVED ON EXTRAPOLATION VECTOR',/)
      CALL OUTPUT(1)
180   GO TO 50
190   IF(G) 90,90,200
200   RETURN
      END
      SUBROUTINE CHCKER
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/EQAL/H,H1,MZ
      COMMON/EXPOPT/ NEXOP1,NEXOP2,XEP1,XEP2
      COMMON/DEVC/NI,NO,NP
      MMZ=1+M+MZ
      DO 5 J=1,N
      DEL(J)=1.2345678
5     CONTINUE
      DO 10 I=1,MMZ
      IN=I-1
      IF (IN) 170,170,180
170   WRITE(NO,001)
001   FORMAT(//,2X,38HVALUES OF OBJECTIVE FUNCTION PARTIALS )
      GO TO 190
180   WRITE (NO,002) IN
002   FORMAT (//,2X,29HVALUES FOR CONSTRAINTS NUMBER ,I2 )
      CALL GRAD1 (IN)
190   WRITE (NO,003)
003   FORMAT (/,2X,25HANALYTICAL FIRST PARTIALS )
      WRITE(NO,004) (J,DEL(J),J=1,N)
004   FORMAT (/,3(2X,4HDEL(,I2,4H) = ,E15.7))
      CALL DIFF1 (IN)
      WRITE(NO,006)
006   FORMAT (/,2X,24HNUMERICAL FIRST PARTIALS )
      WRITE (NO,004) (J,DEL(J),J=1,N)
10    CONTINUE
      IF (NEXOP1.LT.4)GO TO 160
      DO 150 I=1,MMZ
      IN=I-1
      IF (IN) 200,200,210
200   WRITE (NO,001)
      GO TO 220
210   WRITE(NO,002) IN
```

```
220     IT=2
        DO 30 K=1,N
        DO 20 J=1,N
20      A(K,J)=0.0
30      CONTINUE
        CALL MATRIX (IN,IT)
        IF (IT.EQ.1) GO TO 150
        DO 50 K=2,N
        KM1=K-1
        DO 40 J=1,KM1
        IF (A(K,J).EQ.0.0) GO TO 40
        NEXOP1=5
        WRITE (NO,007) K,J
007     FORMAT (/,2X,2HA(,I2,1H,,I2,10H) .NE. 0.0)
        GO TO 60
40      CONTINUE
50      CONTINUE
60      WRITE(NO,009)
009     FORMAT(/,2X,26HANALYTICAL SECOND PARTIALS )
        DO 90 K=1,N
        DO 70 J=K,N
        IF (A(K,J).NE.0.0) GO TO 80
70      CONTINUE
80      WRITE(NO,008) (K,J,A(K,J),J=1,N)
008     FORMAT (/,3(2X,2HA(,I2,1H,,I2,4H) = ,E12.6))
90      CONTINUE
        DO 110 K=1,N
        DO 100 J=1,N
100     A(K,J)=0
110     CONTINUE
        WRITE (NO,011)
011     FORMAT (/,2X,25HNUMERICAL SECOND PARTIALS )
        CALL DIFF2 (IN)
        DO 140 K=1,N
        DO 120 J=K,N
        IF (A(K,J).NE.0) GO TO 130
120     CONTINUE
        GO TO 140
130     WRITE(NO,008) (K,J.A(K,J),J=1,N)
140     CONTINUE
150     CONTINUE
160     CONTINUE
        RETURN
        END
        SUBROUTINE CONVRG(N1)
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
```

```
      COMMON /OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/ F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON /EXPOPT/ NEXOP1,NEXOP2,XEP1,XEP2
      COMMON/TSW/NSWW
      COMMON /DEVC/NI,NO,NP
      NSWW=1
      N1=2
      IF (NT8.LE.1)Q1=PO
      NT8=2
      IF (MN.LE.1)Q1=PO
      GO TO (10,20,30),NT9
10    IF (ABS(DOTT).LT.EPSI)GO TO70
      GO TO 40
20    IF (ABS(DOTT).LT.(P1-PO)/5.0) GO TO 70
      GO TO 40
30    IF (ADELX.LT.EPSI) GO TO 70
40    GO TO (50,60),NSWW
50    IF (MN.LE.1) RETURN
      IF(PO+XEP2.LT.Q1)GO TO 75
      WRITE(NO,80)
80    FORMAT (/,2X, 'ROUND OFF ERRORS PREVENT MORE ACCURATE DETERMI
     1NATION',/,2X,'OF THE MINIMUM OF THIS SUBPROBLEM',/)
      GO TO 70
60    CALL PUNCH
      WRITE (NO,90)
90    FORMAT (///,10X,'***** TIME LIMIT CALLING EXIT FROM COVRG ***
     1**',/)
      N1=3
      RETURN
70    N1=1
75    Q1=PO
      RETURN
      END
      SUBROUTINE DIFF2(IN)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/EXPOPT/NEXOP1,NEXOP2,XEP1,XEP2
      COMMON/STIRX/XSTR(100),XSSS(100),DDLL(100)
      DO 10 J=1,N
10    XSSS(J)=X(J)
      DO 50 J=1,N
      IF (J.EQ.1)GO TO 20
      JM1=J-1
```

```
      X(JM1)=XSSS(JM1)
20    X(J)=XSSS(J)+XEP1
      CALL GRAD1 (IN)
      DO 30 I=1,N
30    DDLL(I)=DEL(I)
      X(J)=XSSS(J)-XEP1
      CALL GRAD1(IN)
      DO 40 I=J,N
40    A(J,I)=(DDLL(I)-DEL(I))/(2.*XEP1)
50    CONTINUE
      X(N)=XSSS(N)
      RETURN
      END
      SUBROUTINE DIFF1(IN)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/EXPOPT/NEXOP1,NEXOP2,XEP1,XEP2
      COMMON/STIRX/XSTR(100),XSSS(100),DDLL(100)
      DO 10 J=1,N
10    XSTR(J)=X(J)
      DO 30 J=1,N
      IF (J.EQ.1) GO TO 20
      JM1=J-1
      X(JM1)=XSTR(JM1)
20    X(J)=XSTR(J)+XEP1
      CALL RESTNT(IN,ZZ2)
      X(J)=XSTR(J)-XEP1
      CALL RESTNT(IN,ZZ1)
30    DEL(J)=(ZZ2-ZZ1)/(2.*XEP1)
      X(N)=XSTR(N)
      RETURN
      END
      SUBROUTINE ESTIM
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON /CONPAR/NF1,NF2,NF3
      COMMON /DEVC/NI,NO,NPP
      CALL STORE
      Z10=RATIO**2
      Z9=RATIO
```

```
      Z1=1.0/Z9+1.0/Z10
      Z2=Z1+1./Z9**3
      Z3=1./Z9**3
      Z4=Z10+Z9
      Z5=Z9**3
      Z6=1.0/((Z10-1.0)*(Z9-1.0))
      Z7=1./Z9
      Z8=1./(Z9-1.)
      RQ=1.0/RHO
      IF(NUMINI-2) 150,80,10
10    WRITE(NO,330)
330   FORMAT(//,2X,'2ND ORDER ESTIMATES',/)
      PO=(PR2-Z4*PR1+Z5*P1)*Z6
      G=(RATIO*G1-GR1)/(RATIO-1.)
      DO 20 I=1,N
20    X(I)=(XR2(I)-Z4*XR1(I)+Z5*X1(I))*Z6
      NP=NPHASE
      NPHASE=4
      CALL EVALU
      NPHASE=NP
      CALL OUTPUT(2)
      GO TO (70,30,70) ,NPHASE
30    DO 50 J=1,M
      IF (RJ(J)) 40,50,50
40    IF (THETAO+RJ(J))70,50,50
50    CONTINUE
      GO TO (70,70,60),NT4
60    CALL FINAL(NF3)
70    CONTINUE
80    WRITE(NO,340)
340   FORMAT (//,2X,'1ST ORDER ESTIMATES',/)
      PO=(Z9*P1-PR1)*Z8
      G=(RATIO*G1-GR1)/(RATIO-1.)
      DO 90 I=1,N
90    X(I)=(Z9*X1(I)-XR1(I))*Z8
      NP=NPHASE
      NPHASE=4
      CALL EVALU
      NPHASE=NP
      CALL OUTPUT(2)
      GO TO (140,100,140) ,NPHASE
100   DO 120 J=1,M
      IF (RJ(J)) 110,120,120
110   IF (RJ(J)+THETAO) 140,120,120
120   CONTINUE
      GO TO (140,130,140),NT4
130   CALL FINAL(NF2)
```

```
140   CONTINUE
150   WRITE(NO,350)
350   FORMAT (//,2X,27HSOLUTION OF THE SUBPROBLEM )
      IF(M) 180,180,160
160   DO 170 J=1,M
170   RJ(J)=RHO/RJ1(J)
180   IF (MZ) 210,210,190
190   DO 200 J=1,MZ
      MNJ=M+J
200   RJ(MNJ)=2.*RJ1(MNJ)*RQ
210   GO TO (220,240) ,NT2
220   DO 230 I=1,N
230   X(I)=RHO/X1(I)
240   CALL OUTPUT(2)
      CALL REJECT
      IF(NUMINI-2)280,300,250
250   GO TO (280,310,260), NT7
260   DO 270 I=1,N
270   DELX(I)=Z1*X1(I)-Z2*XR1(I)+Z3*XR2(I)
280   PR2=PR1
      GR2=GR1
      PR1=P1
      GR1=G1
      DO 290 I=1,N
      XR2(I)=XR1(I)
290   XR1(I)=X1(I)
      RETURN
300   GO TO (280,310,310),NT7
310   DO 320 I=1,N
320   DELX(I)=(X1(I)-XR1(I))*Z7
      GO TO 280
      END
      SUBROUTINE EVALU
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      H=0.0
      RSIGMA=0.0
      F=0.0
      NSATIS=2
      GO TO (10,100,190,200),NPHASE
```

```
10     GO TO (20,40),NT2
20     DO 30 I=1,N
       IF (X(I))260,260,30
30     RSIGMA=RSIGMA-RHO*DLOG(X(I))
40     IF (M.EQ.0) GO TO 90
       DO 80 J=1,M
       CALL RESTNT (J,RJ(J))
       IF (RJ1(J).LE.0.0) GO TO 50
       IF(RJ(J).GT.0.0)GO TO 60
       GO TO 260
50     IF (RJ(J).GT.0.0) GO TO 70
       F=F-RJ(J)
       GO TO 80
60     RSIGMA=RSIGMA-RHO*DLOG(RJ(J))
       GO TO 80
70     NSATIS=1
       RSIGMA=RSIGMA-RHO*DLOG(RJ(J))
80     CONTINUE
90     CONTINUE
       PO=F+RSIGMA
       G=F-RHO*FLOAT(M)
       IF (NT2.EQ.1)G=G-RHO*FLOAT(N)
       RETURN
100     GO TO (110,130), NT2
110     DO 120 I=1,N
        IF(X(I))260,260,120
120     RSIGMA=RSIGMA-RHO*DLOG(X(I))
130     IF (M.EQ.0) GO TO 150
        DO 140 J=1,M
        CALL RESTNT(J,RJ(J))
        IF (RJ(J).LE.0.0) GO TO 260
        RSIGMA=RSIGMA-RHO*DLOG(RJ(J))
140     CONTINUE
150      CONTINUE
        CALL RESTNT(0,F)
        IF(MZ) 180,180,160
160     DO 170 I=1,MZ
        J=I+M
        CALL RESTNT (J,RJ(J))
        H=H+(RJ(J))**2
170     CONTINUE
        H=H/RHO
180     PO=RSIGMA+H
        PO=F+PO
        G=2.*H-RHO*FLOAT(M)
        G=G+F
        IF (NT2.EQ.1) G=G-RHO*FLOAT(N)
```

```
      RETURN
190   RETURN
200   CONTINUE
      IF (M.EQ.0) GO TO 220
      DO 210 I=1,M
      CALL RESTNT (I,RJ(I))
210   CONTINUE
220   CALL RESTNT (0,F)
      IF (MZ) 250,250,230
230   DO 240 I=1,MZ
      KZ=M+I
240   CALL RESTNT(KZ,RJ(KZ))
250   RETURN
260   NSATIS=3
      PO=10.E35
      RETURN
      END
      SUBROUTINE FEAS
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON/DEVC/NI,NO,NP
      NPHASE=1
      GO TO (10,50),NT2
10    NFIX=1
      DO 30 I=1,N
      IF(X(I)) 20,20,30
20    NFIX=2
      X(I)=1.E-05
30    CONTINUE
      GO TO (50,40), NFIX
40    NPHASE=4
      CALL EVALU
      NPHASE=1
      WRITE(NO,130)
130   FORMAT(//,2X,'MADE VARIABLES WHICH VIOLATED NON NEGATIVE
     1CONSTRAINTS SLIGHTLY POSITIVE',/)
      CALL OUTPUT (2)
50    IF (M)90,90,60
60    DO 70 I=1,M
      IF (RJ(I)) 100,100,70
70    CONTINUE
```

```
80     CALL TIMEC
       WRITE(NO,140)
140    FORMAT (//,2X,'THE FEASIBLE STARTING POINT AND VALUES',/)
       G=0.0
       CALL RESTNT(0,F)
       CALL OUTPUT(2)
90     RETURN
100    CALL BODY
       IF(NPHASE.EQ.5) RETURN
       DO 110 I=1,M
       IF(RJ(I)) 120,120,110
110    CONTINUE
       GO TO 80
120    WRITE(NO,150)
150    FORMAT(/////,2X,'THIS PROBLEM POSSESSES NO FEASIBLE STARTING
     1POINT WILL LOOK FOR DATA TO NEXT PROBLEM')
       NPHASE=5
       GO TO 90
       END
       SUBROUTINE FINAL (N2)
       IMPLICIT REAL*8(A-H,O-Z)
       COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
       COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
       COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
       COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
       GO TO (10,20,30),NT5
10     EPSIL=ABS(F/G-1.)
       IF(EPSIL-THETAO) 50,50,70
20     IF (ABS(RSIGMA)-THETAO) 50,50,70
30     IF(NUMINI-1) 50,40,40
40     PEST=PR1-(PR1-PO)/(1.-1./SQRT(RATIO))
       EPSIL=ABS(PEST/G-1.)
       IF (EPSIL-THETAO) 50,70,70
50     N2=1
       GO TO (80,60),NT6
60     CALL PUNCH
       GO TO 80
70     N2=2
80     RETURN
       END
       SUBROUTINE GRAD1(IN)
       IMPLICIT REAL*8(A-H,O-Z)
       COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
       CALL DIFF1 (IN)
```

```
      RETURN
      END
      SUBROUTINE INVERS(NSME)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON /EXPOPT/NEXOP1,NEXOP2,XEP1,XEP2
      COMMON /DEVC/NI,NO,NP
      DIMENSION B(100)
      EQUIVALENCE (B,DELX)
      GO TO (20,170),NSME
20    NINV=0
      IF (A(1,1)) 40,30,50
30    NINV=1
      GO TO 70
40    NINV=1
50    A(1,1)=1./A(1,1)
      DO 60 I=2,N
60    A(1,I)=A(1,I)*A(1,1)
70    DO 160 J=2,N
      JM1=J-1
      T=0
      DO 90 I=1,JM1
      IF(A(I,J)) 80,90,80
80    T=T+A(J,I)*A(I,J)
90    CONTINUE
      A(J,J)=A(J,J)-T
      IF(A(J,J)) 110,100,120
100   NINV=NINV+1
      GO TO 170
110   NINV=NINV+1
120   A(J,J)=1./A(J,J)
      IF(J.EQ.N) GO TO 170
      JP1=J+1
      DO 150 L=JP1,N
      T=0
      DO 140 I=1,JM1
      IF(A(I,J)) 130,140,130
130   T=T+A(L,I)*A(I,J)
140   CONTINUE
      A(L,J)=A(L,J)-T
      A(J,L)=A(L,J)*A(J,J)
```

```
150     CONTINUE
160     CONTINUE
170     CONTINUE
        IF (NINV) 180,180,290
180     B(1)=B(1)*A(1,1)
        DO 210 J=2,N
        T=0
        JM1=J-1
        DO 200 I=1,JM1
        IF(A(J,I)) 190,200,190
190     T=T+A(J,I)*B(I)
200     CONTINUE
        B(J)=(B(J)-T)*A(J,J)
210     CONTINUE
        DO 240 I=1,NM1
        NMK=N-I
        DO 230 J=1,I
        L=NP1-J
        IF (A(NMK,L)) 220,230,220
220     B(NMK)=B(NMK)-A(NMK,L)*B(L)
230     CONTINUE
240     CONTINUE
250     GO TO (280,260),NT3
260     WRITE (NO,430)
430     FORMAT (//,2X,12HDEL P VECTOR  )
        WRITE (NO,420) (I,DELXO(I),I=1,N)
420     FORMAT (/,3(2X,4HDEL(,I2,4H) = ,E16.8))
270     WRITE(NO,440)
440     FORMAT (//,2X,'SECOND ORDER MOVE VECTOR',/)
        WRITE (NO,420)(I,DELX(I),I=1,N)
280     RETURN
290     CONTINUE
        DO 350 II=1,N
        I=N-II+1
        IF(A(I,I)) 310,300,320
300     B(I)=0.0
        GO TO 350
310     B(I)=1.0
        GO TO 330
320     B(I)=0.0
330     IP1=I+1
        IF (IP1.GT.N) GO TO 350
        DO 340 J=IP1,N
340     B(I)=B(I)-A(I,J)*B(J)
350     CONTINUE
        GO TO 360
360     ZC2=0.0
```

```
      DO 370 I=1,N
370   ZC2=ZC2+DELXO(I)*B(I)
      IF (ZC2) 380,400,400
380   DO 390 I=1,N
390   B(I)=-B(I)
400   WRITE (NO,450)
450   FORMAT(//,2X,23HORTHOGONAL MOVE VECTORS )
      IF(NEXOP2.NE.2) GO TO 250
      DO 410 K=1,N
410   B(K)=B(K)+DELXO(K)
      GO TO 250
      END
      SUBROUTINE OUTPUT(K)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON/DEVC/NI,NO,NP
      NZ=M+MZ
      GO TO (10,20),K
10    WRITE(NO,001) NTCTR
001   FORMAT(//,2X,'VALUES AT POINT NUMBER',I2)
      WRITE (NO,002) NPHASE,RHO,RSIGMA
002   FORMAT (/,2X,'PHASE = ',I1,4X,'RHO = ',E16.8,4X,'RSIGMA=',
     1E16.8)
20    WRITE (NO,003) F,PO,G
003   FORMAT (/,2X,4HF = ,E16.8,4X,4HP = ,E16.8,4X,4HG = ,E16.8 )
      WRITE (NO,004)
004   FORMAT (/,2X,18HVALUES OF X VECTOR )
      WRITE(NO,005) (J,X(J), J=1,N)
005   FORMAT (/,3(2X,2HX(,I2,4H) = ,E16.8))
      WRITE(NO,006)
006   FORMAT(/,2X,25HVALUES OF THE CONSTRAINTS )
      GO TO (30,40),NT2
30    WRITE (NO,007)
007   FORMAT(/,2X,'NOT INCLUDING THE NON-NEGATIVE CONSTRAINTS',/)
      WRITE (NO,008) (I,RJ(I),I=1,NZ)
008   FORMAT (/,3(2X,2HG(,I2,4H) = ,E16.8))
      GO TO 50
40    WRITE (NO,008) (I,RJ(I),I=1,NZ)
50    RETURN
      END
```

```
      SUBROUTINE OPT
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON/DEVC/NI,NO,NP
      KSW=1
      N405=1
      P31=PO
      P31=PO
      ISW=1
      DOTT=0
      DO 10 J=1,N
10    DOTT=DOTT+DELX(J)*DELXO(J)
      GO TO 40
20    DO 30 I=1,N
30    DELX(I)=-DELX(I)
40    CONTINUE
      N404=0
      MN=MN+1
      NTCTR=NTCTR+1
      DO 50 I=1,N
50    X2(I)=X(I)
      PX1=PO
      N401=0
60    N401=N401+1
      DO 70 I=1,N
70    X(I)=X2(I)+DELX(I)
      CALL EVALU
      GO TO (540,90,80),NSATIS
80    PX2=10.E35
      GO TO 100
90    CONTINUE
      PX2=PO
      IF(PX1-PX2)100,100,150
100   IF (N401-2) 130,110,110
110   DO 120 I=1,N
120   X1(I)=X(I)
      P1=PX2
      GO TO 430
130   DO 140 I=1,N
140   X3(I)=X2(I)
      PREV3=PX1
      GO TO 180
```

```
150    DO 160 I=1,N
       X3(I)=X2(I)
       X2(I)=X(I)
160    DELX(I)=1.61803399*DELX(I)
       PREV3=PX1
       PX1=PX2
       GO TO 60
170    P0=1.E36
       N404=N404+1
180    DO 190 I=1,N
190    X1(I)=X(I)
       P1=P0
       DO 200 I=1,N
       X(I)=.38196601*(X1(I)-X3(I))+X3(I)
200    X2(I)=X(I)
       CALL EVALU
       GO TO (540,270,210),NSATIS
210    IF (N404.LT.30) GO TO 170
       IF (N404.GT.100) GO TO 240
220    DO 230 I=1,N
       IF (ABS(ABS(X3(I)/X1(I))-1.).GT.1.E-7) GO TO 170
230    CONTINUE
240    GO TO (250,260),N405
250    N405=2
       NTCTR=NTCTR-1
       MN=MN-1
       GO TO 20
260    WRITE (7,580)
580    FORMAT(//,2X,'CAN T FIND A FEASIBLE POINT THAT GIVES A LOWER
     1VALUE OF THE FUNCTION REENTER STARTING POINTS PLEASE',/)
       CALL TIMEC
       CALL OUTPUT(1)
       CALL REJECT
       STOP
270    CONTINUE
       N404=0
       PX1=P0
       DO 280 I=1,N
280    X(I)=0.38196601*(X1(I)-X2(I))+X2(I)
       CALL EVALU
       GO TO (540,290,220),NSATIS
290    PX2=P0
       N401=1
300    N401=N401+1
       IF (N401-25) 340,310,310
310    KSW=2
       IF(N401-40) 320,460,460
```

```
320    DO 330 I=1,N
       IF (ABS(X2(I)/X(I)-1.0).GE.1.E-7) GO TO 340
330    CONTINUE
       GO TO 460
340    IF(ABS(PX1/PX2-1.).LE.1.E-7) GO TO 460
       IF(PX1-PX2)350,460,400
350    DO 360 I=1,N
360    X1(I)=X(I)
       P1=PX2
       DO 370 I=1,N
370    X(I)=0.38196601*(X1(I)-X3(I))+X3(I)
       CALL EVALU
       GO TO (540,380,170), NSATIS
380    CONTINUE
       PX2=PX1
       PX1=PO
       DO 390 I=1,N
       XX=X2(I)
       X2(I)=X(I)
390    X(I)=XX
       GO TO 300
400    IF(PREV3-PX2) 350,350,410
410    DO 420 I=1,N
       X3(I)=X2(I)
420    X2(I)=X(I)
       PREV3=PX1
       PX1=PX2
430    DO 440 I=1,N
440    X(I)=0.38196601*(X1(I)-X2(I))+X2(I)
       CALL EVALU
       GO TO (540,450,170), NSATIS
450    CONTINUE
       PX2=PO
       GO TO 300
460    DO 470 I=1,N
       DELXO(I)=X(I)
       X(I)=(DELXO(I)+X2(I))*0.5
470    CONTINUE
       CALL EVALU
       GO TO (480,490),KSW
480    IF (ABS (PO/PX1-1.).GT.1.E-7) GO TO 520
490    GO TO (500,510),ISW
500    IF (PO.LT.P31) GO TO 510
       ISW=2
       GO TO 20
510    RETURN
520    DO 530 I=1,N
```

```
530   X(I)=DELXO(I)
      GO TO 350
540   DO 550 I=1,M
      IF (RJ(I)) 560,560,550
550   CONTINUE
      NSATIS=4
      RETURN
560   MN=0
      DO 570 I=1,M
570   RJ1(I)=RJ(I)
888   RETURN
      END
      SUBROUTINE PUNCH
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON/EXPOPT/NEXOP1,NEXOP2,XEP1,XEP2
      COMMON/DEVC/NI,NO,NP
      T=60.0
      NT1=3
      RETURN
      END
      SUBROUTINE PEVALU
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      H=0.0
      RSIGMA=0.0
      GO TO (10,30),NT2
10    DO 20 I=1,N
20    RSIGMA=RSIGMA-RHO*DLOG(X(I))
30    GO TO (40,50,150),NPHASE
40    F=0.0
50    IF (M) 100,100,60
60    DO 90 J=1,M
```

```
      IF(RJ(J))80,80,70
70    RSIGMA=RSIGMA-RHO*DLOG(RJ(J))
      GO TO 90
80    F=F-RJ(J)
90    CONTINUE
100   CONTINUE
      IF(MZ) 140,140,110
110   GO TO (140,140,150),NPHASE
120   DO 130 I=1,MZ
      K=M+I
130   H=H+RJ(K)**2
      H=H/RHO
140   HS=H+RSIGMA
      PO=F+HS
      HMS=2.*H-RHO*FLOAT(M)
      G=F+HMS
      IF(NT2.EQ.1) G=G-RHO*FLOAT(N)
150   RETURN
      END
      SUBROUTINE RHOCOM
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      GO TO (110,50,10,190), NT1
10    RHO=RHOIN
20    IF(RHO) 30,30,40
30    RHO=1
40    RETURN
50    NPAR1=1
60    RHO=1
      CALL GRAD(2)
      DO 70 I=1,N
70    PGRAD(I)=DELXO(I)
      RHO=2
      CALL GRAD(2)
      DO 80 I=1,N
      DELXO(I)=DELXO(I)-PGRAD(I)
80    PGRAD(I)=PGRAD(I)-DELXO(I)
      GO TO (90,130),NPAR1
90    DOT1=0
      DOT2=0
      DO 100 I=1,N
```

```
      DOT1=DOT1+DELXO(I)*PGRAD(I)
100   DOT2=DOT2+DELXO(I)**2
      RHO=ABS(DOT1/DOT2)
      GO TO 20
110   NPAR2=1
120   NPAR1=2
      GO TO 60
130   RHO=1
      CALL SECORD(2)
      DO 140 I=1,N
140   DELX(I)=PGRAD(I)
      CALL INVERS(1)
      DO 150 I=1,N
      X1(I)=DELX(I)
150   DELX(I)=DELXO(I)
      CALL SECORD(2)
      CALL INVERS(1)
      DO 160 I=1,N
160   XR2(I)=DELX(I)
      GO TO (170,200),NPAR2
170   DOT1=0.
      DOT2=0.
      DO 180 I=1,N
      DOT1=DOT1+PGRAD(I)*X1(I)
180   DOT2=DOT2+DELXO(I)*XR2(I)
      RHO=SQRT(ABS(DOT1/DOT2))
      GO TO 20
190   NPAR2=2
      GO TO 120
200   DOT1=0.0
      DOT2=0.0
      DO 210 I=1,N
      DOT1=X1(I)**2+DOT1
210   DOT2=X1(I)*XR2(I)+DOT2
      RHO=ABS(DOT1/DOT2)
      GO TO 20
      END
      SUBROUTINE SECORD(IS)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100).N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
```

```
      DO 10 I=1,N
      DO 10 J=1,N
10    A(I,J)=0.
      GO TO (230,20),IS
20    DO 30 I=1,N
      DO 30 J=1,I
      A(I,J)=0.0
30    CONTINUE
      GO TO (40,60), NT2
40    DO 50 I=1,N
50    A(I,I)=RHO/X(I)**2
60    CONTINUE
      IF (M.LE.0) GO TO 130
      DO 120  IN=1,M
      IF (RJ(IN)) 120,120,70
70    CALL GRAD1(IN)
      TT=RHO/RJ(IN)**2
      DO 110 I=1,N
      IF (DEL(I)) 80,110,80
80    T=TT*DEL(I)
      DO 100 J=1,I
      IF(DEL(J)) 90,100,90
90    A(I,J)=A(I,J)+T*DEL(J)
100   CONTINUE
110   CONTINUE
120   CONTINUE
130   IF (MZ) 210,210,140
140   GO TO (210,150,230),NPHASE
150   RQ=2./RHO
      DO 200 JJ=1,MZ
      IN=M+JJ
      CALL GRAD1(IN)
      DO 190 I=1,N
      IF (DEL(I)) 160,190,160
160   T=RQ*DEL(I)
      DO 180 J=1,I
      IF (DEL(J)) 170,180,170
170   A(I,J)=A(I,J)+T*DEL(J)
180   CONTINUE
190   CONTINUE
200   CONTINUE
210   DO 220 I=1,N
      DIAG(I)=A(I,I)
220   A(I,I)=0.
230   GO TO (240,510,520),NT10
240   IF (M.LE.0) GO TO 340
      DO 330 IN=1,M
```

```
      LORN=2
      CALL MATRIX(IN,LORN)
      IF (LORN.LT.2) GO TO 330
      IF (RJ(IN).GT.0.0) GO TO 280
      DO 260 I=2,N
      IM1=I-1
      DO 260 J=1,IM1
      IF(A(J,I)) 250,260,250
250   A(I,J)=A(I,J)-A(J,I)
      A(J,I)=0.
260   CONTINUE
      DO 270 I=1,N
      DIAG(I)=DIAG(I)-A(I,I)
270   A(I,I)=0.0
      GO TO 330
280   T=-RHO/RJ(IN)
      DO 300 I=2,N
      IM1=I-1
      DO 300 J=1,IM1
      IF (A(J,I)) 290,300,290
290   A(I,J)=A(I,J)+T*A(J,I)
      A(J,I)=0.
300   CONTINUE
      DO 320 I=1,N
      IF (A(I,I)) 310,320,310
310   DIAG(I)=DIAG(I)+T*A(I,I)
      A(I,I)=0
320   CONTINUE
330   CONTINUE
340   CONTINUE
      GO TO (520,350,520) ,NPHASE
350   IF (MZ.EQ.0) GO TO 420
      IF (NT10.GE.2) GO TO 420
      DO 410 II=1,MZ
      IN=M+II
      LORN=2
      CALL MATRIX (IN,LORN)
      IF(LORN.LT.2) GO TO 410
      T=2.*RJ(IN)/RHO
      DO 380 I=2,N
      IM1=I-1
      DO 370 J=1,IM1
      IF(A(J,I)) 360,370,360
360   A(I,J)=A(I,J)+T*A(J,I)
      A(J,I)=0.0
370   CONTINUE
380   CONTINUE
```

```
      DO 400  I=1,N
      IF(A(I,I)) 390,400,390
390   DIAG(I)=DIAG(I)+T*A(I,I)
      A(I,I)=0.0
400   CONTINUE
410   CONTINUE
420   LLL=2
      CALL MATRIX(0,LLL)
      IF(LLL.LT.2) GO TO 490
      DO 440 I=2,N
      IM1=I-1
      DO 440 J=1,IM1
      IF(A(J,I)) 430,440,430
430   A(I,J)=A(I,J)+A(J,I)
440   A(J,I)=A(I,J)
      DO 470 I=1,N
      IF(A(I,I)) 450,460,450
450   A(I,I)=DIAG(I)+A(I,I)
      GO TO 470
460   A(I,I)=DIAG(I)
470   CONTINUE
480   RETURN
490   DO 500 I=1,N
      A(I,I)=DIAG(I)
      DO 500 J=1,N
500   A(I,J)=A(J,I)
      GO TO 480
510   GO TO (520,350,350),NPHASE
520   DO 530 I=2,N
      IM1=I-1
      DO 530 J=1,IM1
530   A(J,I)=A(I,J)
      DO 540 I=1,N
540   A(I,I)=DIAG(I)
      GO TO 480
      END
      SUBROUTINE XMOVE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      COMMON/EXPOPT/NEXOP1,NEXOP2,XEP1,XEP2
      COMMON/XVE/SIG(100),YY(100),XXX(100),DELL(100)
      GO TO (10,10,180,30),NEXOP2
10    CALL GRAD(1)
```

```
      CALL SECORD(1)
      DO 20 I=1,N
20    DELX(I)=DELXO(I)
      CALL INVERS(1)
      CALL STORE
      CALL OPT
      RETURN
30    CALL GRAD(2)
      IF(MN.NE.0.0) GO TO 70
40    IREP=0
      IT=0
      DO 50 I=1,N
      DO 50 J=1,N
50    A(I,J)=0.0
      DO 60 I=1,N
60    A(I,I)=1.0
70    DO 80 I=1,N
80    DELX(I)=DELXO(I)
      IF(IREP.GT.N) GO TO 40
      IF(IT.EQ.0) GO TO 130
      DO 90 I=1,N
      SIG(I)=X(I)-XXX(I)
90    YY(I)=DELL(I)-DELXO(I)
      DO 100 I=1,N
      DELX(I)=0.0
      DO 100 J=1,N
100   DELX(I)=DELX(I)+A(I,J)*YY(J)
      ZCON=0.0
      DO 110 I=1,N
110   ZCON=ZCON+YY(I)*(SIG(I)-DELX(I))
      IF (ZCON.EQ.0.0) GO TO 130
      IREP=IREP+1
      ZC=1.0/ZCON
      DO 120 I=1,N
      T1=ZC*(SIG(I)-DELX(I))
      DO 120 J=1,N
      A(I,J)=A(I,J)+T1*(-DELX(J)+SIG(J))
120   A(J,I)=A(I,J)
130   DO 140 I=1,N
      XXX(I)=X(I)
140   DELL(I)=DELXO(I)
      DO 150 I=1,N
      DELX(I)=0.0
      DO 150 J=1,N
150   DELX(I)=DELX(I)+A(I,J)*DELXO(J)
      ZC1=0.0
      DO 160 I=1,N
```

```
160     ZC1=DELX(I)**2+ZC1
        ZC1=SQRT(ZC1)
        DO 170 I=1,N
170     DELX(I)=DELX(I)/ZC1
        CALL STORE
        CALL OPT
        IT=IT+1
        RETURN
180     CONTINUE
        CALL GRAD(2)
        DO 190 I=1,N
190     DELX(I)=DELXO(I)
        CALL STORE
        CALL OPT
        RETURN
        END
        SUBROUTINE STORE
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
        COMMON /EQAL/H,H1,MZ
        COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
        COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
      1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
      2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
      3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
        DO 10 I=1,N
10      X1(I)=X(I)
        MMZ=M+MZ
        DO 20 J=1,MMZ
20      RJ1(J)=RJ(J)
        P1=PO
        F1=F
        G1=G
        RSIG1=RSIGMA
        H1=H
        RETURN
        END
        SUBROUTINE TCHECK
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
        COMMON/TSW/NSWW
        COMMON/TMX/TMO,EXT,EXT90
        COMMON/DEVC/NI,NO,NP
        RETURN
        END
        SUBROUTINE REJECT
        IMPLICIT REAL*8(A-H,O-Z)
```

```
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      DO 10 I=1,N
10    X(I)=X1(I)
      MMZ=M+MZ
      DO 20 J=1,MMZ
20    RJ(J)=RJ1(J)
      PO=P1
      RSIGMA=RSIG1
      G=G1
      F=F1
      H=H1
      RETURN
      END
      SUBROUTINE GRAD(IS)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100).N,M,MN,NP1,NM1
      COMMON /EQAL/H,H1,MZ
      COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
      GO TO (10,30),IS
10    DO 20 I=1,N
      DO 20 J=1,I
20    A(I,J)=0
30    DO 40 I=1,N
40    DELXO(I)=0
      GO TO (50,80),NT2
50    DO 70 I=1,N
      DELXO(I)=-RHO/X(I)
      GO TO (60,70),IS
60    A(I,I)=(-DELXO(I)/X(I))
70    CONTINUE
80    CONTINUE
      IF(M.LE.0) GO TO 180
      DO 170 K=1,M
      CALL GRAD1(K)
      IF (RJ(K).GT.0.0) GO TO 110
      DO 100 I=1,N
```

```
      IF(DEL(I)) 90,100,90
90    DELXO(I)=DELXO(I)-DEL(I)
100   CONTINUE
      GO TO 170
110   TT=RHO/RJ(K)
      DO 160 I=1,N
      IF(DEL(I)) 120,160,120
120   T=TT*DEL(I)
      DELXO(I)=DELXO(I)-T
      GO TO (130,160),IS
130   T=T/RJ(K)
      DO 150 JJ=1,I
      IF (DEL(JJ)) 140,150,140
140   A(I,JJ)=A(I,JJ)+T*DEL(JJ)
150   CONTINUE
160   CONTINUE
170   CONTINUE
180   IF(MZ.LE.0)GO TO 250
      GO TO (250,190,250), NPHASE
190   RQ=2./RHO
      DO 240 J=1,MZ
      K=M+J
      CALL GRAD1(K)
      TT=RQ*RJ(K)
      DO 230 I=1,N
      IF (DEL(I).EQ.0.0) GO TO 230
      DELXO(I)=DELXO(I)+DEL(I)*TT
      GO TO (200,230),IS
200   T=RQ*DEL(I)
      DO 220  JJ=1,I
      IF(DEL(JJ)) 210,220,210
210   A(I,JJ)=A(I,JJ)+T*DEL(JJ)
220   CONTINUE
230   CONTINUE
240   CONTINUE
250   GO TO (260,280),IS
260   DO 270 I=1,N
270   DIAG(I)=A(I,I)
280   GO TO (290,330,290),NPHASE
290   DO 300 I=1,N
300   DELXO(I)=-DELXO(I)
310   ADELX=0
      DO 320 I=1,N
320   ADELX=ADELX+DELXO(I)**2
      ADELX=SQRT(ADELX)
      RETURN
330   CALL GRAD1(0)
```

```
      DO 340 I=1,N
340   DELXO(I)=-DELXO(I)-DEL(I)
      GO TO 310
      END
      SUBROUTINE MATRIX(J,L)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      CALL DIFF2(J)
      RETURN
      END
      SUBROUTINE RESTNT(IN,VAL)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/EXTM/M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14,M15
      COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
      IF(MOPT.EQ.5) GO TO 4
      IF(MOPT.EQ.2) GO TO 3
      IF(MOPT.EQ.4) GO TO 2
      IF(MOPT.EQ.1) GO TO 1
      CALL RESAND(IN,VAL)
      RETURN
1     CALL RESNDS(IN,VAL)
      RETURN
2     CALL RESUMD(IN,VAL)
      RETURN
3     CALL RESAND(IN,VAL)
      RETURN
4     CALL RESEXM(IN,VAL)
      RETURN
      END
      SUBROUTINE RESNDS(IN,VAL)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
      IF(NOBJ.EQ.3) GO TO 555
      IF(IN) 123,123,124
123   VAL=NNP*(ALPHAO*X(1)**(-1)*X(2)**(-1)+ALPHA1*X(1)**BETA1*X(2)**
     aBETA2*BOU(15)**BETA4*(2.718)**(BETA3*X(2))+CM)
      GO TO 99
```

```
124     GO TO (1,2,3,4,5,6,7,8,9,10,11),IN
C.SPEED
1       VAL=BOU(1)-X(1)
        GO TO 99
2       VAL=X(1)-BOU(2)
        GO TO 99
C.FEED
3       VAL=BOU(3)-X(2)
        GO TO 99
4       VAL=X(2)-BOU(4)
        GO TO 99
C.POWER
5      VAL=BOU(7)-(C3*X(1)**(A1)*X(2)**A2*BOU(15))*X(1)
        GO TO 99
C.CUTTING FORCE
6       VAL=BOU(8)-C2*X(1)**(A1)*X(2)**A2*BOU(15)
        GO TO 99
C.CUTTING TEMPERATURE
7       VAL=BOU(9)-C5*X(1)**A7*X(2)**A8*BOU(15)**A9
        GO TO 99
C.TOOL LIFE
8      VAL=BOU(10)-(C1*X(1)**(B1)*2.718**(B3*X(2))*BOU(15)
     @**B4)
        GO TO 99
9      VAL=(C1*X(1)**(B1)*2.718**(B3*X(2))*BOU(15)
     @**B4)-BOU(11)
        GO TO 99
C.SURFACE FINISH
10      VAL=BOU(12)-(C6*X(1)**(A10)*X(2)**A11*BOU(15)**A12)
        GO TO 99
C.STABILITY
11      VAL=C4*X(1)**A5*X(2)**A6-BOU(13)
        GO TO 99
555     IF(IN) 523,523,524
523     VAL=-(10E-6*X(1)*X(2)*BOU(15))
        GO TO 99
524     GO TO (1,2,3,4,5,6,7,8,9,10,11),IN
99      RETURN
        END
        SUBROUTINE RESUMD(IN,VAL)
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
        COMMON/BB/BOU(30)
        COMMON/BBC/NNP,MOPT,NOBJ
        COMMON/EXTM/M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14,M15
        COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
```

```
     3CM
       DIMENSION V(20),VV(20),VV1(20)
       IF(NOBJ.EQ.3) GO TO 525
       IF(IN) 123,123,124
123   KK=0
      VAL=0.0
      DO 555 II=1,NNP
      I=II+KK
      J=II+KK+1
      K=II+KK+2
      V(II)=ALPHA0*X(I)**(-1)*X(J)**(-1)+ALPHA1*X(I)**
     aBETA1*X(J)** BETA2*X(K)**BETA4*(2.718)**(BETA3*X(J))+CM
      KK=KK+2
      VAL=V(II)+VAL
555   CONTINUE
      RETURN
124    GO TO (111,222,333,4441,5551,6661,7771,8881,9991,9995,909,
     a919,929,939,949,969),NNP
126    GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15),MM
127    GO TO (1,2,3,4,5,6,7,8,9,17,18,16,13,14,15),MM
C.SPEED
1      VAL=BOU(1)-X(I)
       GO TO 99
2      VAL=X(I)-BOU(2)
       GO TO 99
C.FEED
3      VAL=BOU(3)-X(J)
       GO TO 99
4      VAL=X(J)-BOU(4)
       GO TO 99
C.DEPTH OF CUT
5      VAL=BOU(5)-X(K)
       GO TO 99
6      VAL=X(K)-BOU(6)
       GO TO 99
C.POWER
7      VAL=BOU(7)-(C3*X(I)**(A1)*X(J)**A2*X(K))*X(I)
       GO TO 99
C.CUTTING FORCE
8      VAL=BOU(8)-C2*X(I)**(A1)*X(J)**A2*X(K)
       GO TO 99
C.CUTTING TEMPERATURE
9      VAL=BOU(9)-C5*X(I)**A7*X(J)**A8*X(K)**A9
       GO TO 99
C.TOOL LIFE FOR FINISHING
10    VAL=BOU(10)-(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)
     a**B4)
```

```
      GO TO 99
11    VAL=(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)
     @**B4)-BOU(11)
      GO TO 99
C.TOOL LIFE FOR ROUGHING
17    VAL=BOU(17)-(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)
     @**B4)
      GO TO 99
18    VAL=(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)
     @**B4)-BOU(18)
      GO TO 99
C.SURFACE FINISH
12    VAL=BOU(12)-(C6*X(I)**(A10)*X(J)**A11*X(K)**A12)
      GO TO 99
16    VAL=BOU(16)-(C6*X(I)**(A10)*X(J)**A11*X(K)**A12)
      GO TO 99
C.STABILITY
13    VAL=C4*X(I)**A5*X(J)**A6-BOU(13)
      GO TO 99
14     VAL=0
       DO 33 II=3,N,3
       IF(II.GT.3) GO TO 733
       VV(II)=BOU(14)-X(II)
       VAL=VV(II)+VAL
       GO TO 33
733    VAL=VAL-X(II)
33     CONTINUE
       RETURN
15     VAL=0
       DO 44 II=3,N,3
       IF(II.GT.3) GO TO 833
       VV1(II)=-BOU(15)+X(II)
       VAL=VV1(II)+VAL
       GO TO 44
833    VAL=VAL+X(II)
44     CONTINUE
       RETURN
111   MM=IN
      I=1
      J=2
      K=3
      GO TO 126
222   IF(IN.LE.M1) GO TO 111
444   MM=IN-M1
      I=4
      J=5
      K=6
```

```
      IF(NNP.GT.2) GO TO 126
      GO TO 127
333   IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
666   MM=IN-M2
      I=7
      J=8
      K=9
      IF(NNP.GT.3) GO TO 126
      GO TO 127
4441  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
5552  MM=IN-M3
      I=10
      J=11
      K=12
      IF(NNP.GT.4) GO TO 126
      GO TO 127
5551  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
6662  MM=IN-M4
      I=13
      J=14
      K=15
      IF(NNP.GT.5) GO TO 126
      GO TO 127
6661  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
7772  MM=IN-M5
      I=16
      J=17
      K=18
      IF(NNP.GT.6) GO TO 126
      GO TO 127
7771  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
8882  MM=IN-M6
```

```
      I=19
      J=20
      K=21
      IF(NNP.GT.7) GO TO 126
      GO TO 127
8881  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
      IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
9992  MM=IN-M7
      I=22
      J=23
      K=24
      IF(NNP.GT.8) GO TO 126
      GO TO 127
9991  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
      IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
      IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
9996  MM=IN-M8
      I=25
      J=26
      K=27
      IF(NNP.GT.9) GO TO 126
      GO TO 127
9995  IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
      IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
      IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
      IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
9997  MM=IN-M9
      I=28
      J=29
      K=30
      IF(NNP.GT.10) GO TO 126
      GO TO 127
```

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909    IF(IN.LE.M1) GO TO 111
       IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
       IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
       IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
       IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
       IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
       IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
       IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
       IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
       IF(IN.GT.M9.AND.IN.LE.M10) GO TO 9997
9998   MM=IN-M10
       I=31
       J=32
       K=33
       IF(NNP.GT.11) GO TO 126
       GO TO 127
919    IF(IN.LE.M1) GO TO 111
       IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
       IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
       IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
       IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
       IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
       IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
       IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
       IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
       IF(IN.GT.M9.AND.IN.LE.M10) GO TO 9997
       IF(IN.GT.M10.AND.IN.LE.M11) GO TO 9998
9999   MM=IN-M11
       I=34
       J=35
       K=36
       IF(NNP.GT.12) GO TO 126
       GO TO 127
929    IF(IN.LE.M1) GO TO 111
       IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
       IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
       IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
       IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
       IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
       IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
       IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
       IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
       IF(IN.GT.M9.AND.IN.LE.M10) GO TO 9997
       IF(IN.GT.M10.AND.IN.LE.M11) GO TO 9998
       IF(IN.GT.M11.AND.IN.LE.M12) GO TO 9999
9099   MM=IN-M12
       I=37
```

```
      J=38
      K=39
      IF(NNP.GT.13) GO TO 126
      GO TO 127
939   IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
      IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
      IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
      IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
      IF(IN.GT.M9.AND.IN.LE.M10) GO TO 9997
      IF(IN.GT.M10.AND.IN.LE.M11) GO TO 9998
      IF(IN.GT.M11.AND.IN.LE.M12) GO TO 9999
      IF(IN.GT.M12.AND.IN.LE.M13) GO TO 9099
959   MM=IN-M13
      I=40
      J=41
      K=42
      IF(NNP.GT.14) GO TO 126
      GO TO 127
949   IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
      IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
      IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
      IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
      IF(IN.GT.M9.AND.IN.LE.M10) GO TO 9997
      IF(IN.GT.M10.AND.IN.LE.M11) GO TO 9998
      IF(IN.GT.M11.AND.IN.LE.M12) GO TO 9999
      IF(IN.GT.M12.AND.IN.LE.M13) GO TO 9099
      IF(IN.GT.M13.AND.IN.LE.M14) GO TO 959
979   MM=IN-M14
      I=43
      J=44
      K=45
      IF(NNP.GT.15) GO TO 126
      GO TO 127
969   IF(IN.LE.M1) GO TO 111
      IF(IN.GT.M1.AND.IN.LE.M2) GO TO 444
      IF(IN.GT.M2.AND.IN.LE.M3) GO TO 666
      IF(IN.GT.M3.AND.IN.LE.M4) GO TO 5552
```

```
      IF(IN.GT.M4.AND.IN.LE.M5) GO TO 6662
      IF(IN.GT.M5.AND.IN.LE.M6) GO TO 7772
      IF(IN.GT.M6.AND.IN.LE.M7) GO TO 8882
      IF(IN.GT.M7.AND.IN.LE.M8) GO TO 9992
      IF(IN.GT.M8.AND.IN.LE.M9) GO TO 9996
      IF(IN.GT.M9.AND.IN.LE.M10) GO TO 9997
      IF(IN.GT.M10.AND.IN.LE.M11) GO TO 9998
      IF(IN.GT.M11.AND.IN.LE.M12) GO TO 9999
      IF(IN.GT.M12.AND.IN.LE.M13) GO TO 9099
      IF(IN.GT.M13.AND.IN.LE.M14) GO TO 959
      IF(IN.GT.M14.AND.IN.LE.M15) GO TO 979
      MM=IN-M15
      I=46
      J=47
      K=48
      GO TO 127
525   IF(IN) 523,523,524
523   KK=0
      VAL=0.0
      DO 565 II=1,NNP
      I=II+KK
      J=II+KK+1
      K=II+KK+2
      V(II)=-(10E-6*X(I)*X(J)*X(K))
      KK=KK+2
      VAL=V(II)+VAL
565   CONTINUE
      RETURN
524     GO TO (111,222,333,4441,5551,6661,7771,8881,9991,9995,909,
     a919,929,939,949,969),NNP
99      RETURN
        END
        SUBROUTINE RESAND(IN,VAL)
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
        COMMON/BB/BOU(30)
        COMMON/BBC/NNP,MOPT,NOBJ
        COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
        IF(NOBJ.EQ.3) GO TO 550
        IF(NNP.GT.1.0) GO TO 991
        IF(NNP.EQ.0.0) GO TO 992
991     IF(IN) 123,123,124
123   VAL=((BOU(15)-X(6))/X(3))*((ALPHAO*X(1)**(-1)*X(2)**(-1)+ALPHA1
     a*X(1)**BETA1*X(2)**
     a(BETA2)*X(3)**(BETA4)*(2.718)**(BETA3*X(2))+CM))+
```

```
    a(ALPHA0*X(4)**(-1)*X(5)**(-1)+ALPHA1*X(4)**BETA1*X(5)**
    a(BETA2)*X(6)**(BETA4)*(2.718)**(BETA3*X(5))+CM)
       GO TO 99
124    GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,27,28
    a,21,22,23,24,25,26),IN
C.SPEED
1      VAL=BOU(1)-X(1)
       GO TO 99
2      VAL=X(1)-BOU(2)
       GO TO 99
3      VAL=BOU(1)-X(4)
       GO TO 99
4      VAL=X(4)-BOU(2)
       GO TO 99
C.FEED
5      VAL=BOU(3)-X(2)
       GO TO 99
6      VAL=X(2)-BOU(4)
       GO TO 99
7      VAL=BOU(3)-X(5)
       GO TO 99
8      VAL=X(5)-BOU(4)
       GO TO 99
C.DEPTH OF CUT
9      VAL=BOU(5)-X(3)
       GO TO 99
10     VAL=X(3)-BOU(6)
       GO TO 99
11     VAL=BOU(5)-X(6)
       GO TO 99
12     VAL=X(6)-BOU(6)
       GO TO 99
C.POWER
13     VAL=BOU(7)-(C3*X(1)**(A1)*X(2)**A2*X(3))*X(1)
       GO TO 99
14    VAL=BOU(7)-(C3*X(4)**(A1)*X(5)**A2*X(6))*X(4)
       GO TO 99
C.CUTTING FORCE
15     VAL=BOU(8)-C2*X(1)**(A1)*X(2)**A2*X(3)
       GO TO 99
16     VAL=BOU(8)-C2*X(4)**(A1)*X(5)**A2*X(6)
       GO TO 99
C.CUTTING TEMPERATURE
17     VAL=BOU(9)-C5*X(1)**A7*X(2)**A8*X(3)**A9
       GO TO 99
18     VAL=BOU(9)-C5*X(4)**A7*X(5)**A8*X(6)**A9
       GO TO 99
```

```
C.TOOL LIFE
19     VAL=BOU(10)-(C1*X(1)**(B1)*2.718**(B3*X(2))*X(3)**B4)
        GO TO 99
20      VAL=(C1*X(1)**(B1)*2.718**(B3*X(2))*
      @X(3)**B4)-BOU(11)
        GO TO 99
C.TOOL LIFE FOR ROUGHING
27     VAL=BOU(17)-(C1*X(4)**(B1)*2.718**(B3*X(5))*X(6)**B4)
        GO TO 99
28      VAL=(C1*X(4)**(B1)*2.718**(B3*X(5))*
      @X(6)**B4)-BOU(18)
        GO TO 99
C.SURFACE FINISH
21      VAL=BOU(12)-(C6*X(1)**(A10)*X(2)**A11*X(3)**A12)
        GO TO 99
22      VAL=BOU(13)-(C6*X(4)**(A10)*X(5)**A11*X(6)**A12)
        GO TO 99
C.STABILITY
23      VAL=X(1)**A5*X(2)-BOU(14)
        GO TO 99
24      VAL=X(4)**A5*X(5)-BOU(14)
        GO TO 99
25       VAL= BOU(15)-X(6)-NNP*X(3)
        GO TO 99
26       VAL=-BOU(16)+((NNP*X(3))+X(6))
        GO TO 99
992     IF(IN) 223,223,224
223    VAL=(ALPHAO*X(1)**(-1)*X(2)**(-1)+ALPHA*X(1)**BETA1*X(2)**
      @(BETA2)*BOU(15)**(BETA4)*(2.718)**(BETA3*X(2))+CM)
        GO TO 99
224     GO TO (31,32,33,34,37,38,39,40,41,42,43),IN
C.SPEED
31      VAL=BOU(1)-X(1)
        GO TO 99
32      VAL=X(1)-BOU(2)
        GO TO 99
C.FEED
33      VAL=BOU(3)-X(2)
        GO TO 99
34      VAL=X(2)-BOU(4)
        GO TO 99
C.POWER
37     VAL=BOU(7)-(C3*X(1)**(A1)*X(2)**A2*BOU(15))*X(1)
        GO TO 99
C.CUTTING FORCE
38      VAL=BOU(8)-C2*X(1)**(A1)*X(2)**A2*BOU(15)
        GO TO 99
```

```
C.CUTTING TEMPERATURE
39      VAL=BOU(9)-C5*X(1)**A7*X(2)**A8*BOU(15)**A9
        GO TO 99
C.SURFACE FINISH
40      VAL=BOU(13)-(C6*X(1)**(A10)*X(2)**A11*BOU(15)**A12)
        GO TO 99
C.TOOL LIFE
42     VAL=BOU(17)-(C1*X(1)**(B1)*2.718**(B3*X(2))*X(3)**B4)
        GO TO 99
43      VAL=(C1*X(1)**(B1)*2.718**(B3*X(2))*
       ∂X(3)**B4)-BOU(18)
        GO TO 99
C.STABILITY
41      VAL=X(1)**2*X(2)-BOU(14)
        GO TO 99
550     IF(NNP.GT.1.0) GO TO 591
        IF(NNP.EQ.0.0) GO TO 592
591     IF(IN) 523,523,524
523     VAL=-(10E-6*(X(1)*X(2)*X(3)+X(4)*X(5)*X(6)))
        GO TO 99
524     GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,27,28
       ∂,21,22,23,24,25,26),IN
592     IF(IN) 243,243,254
243     VAL=-(10E-6*(X(1)*X(2)*X(3)))
        GO TO 99
254     GO TO (31,32,33,34,37,38,39,40,41,42,43),IN
99      RETURN
        END
        SUBROUTINE RESEXM(IN,VAL)
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON/SHARE/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
        COMMON/BB/BOU(30)
        COMMON/BBC/NNP,MOPT,NOBJ
C. EXAMPLE PROBLEM KUESTER AND MIZE
  992     IF(IN) 223,223,224
  223     VAL=X(1)*X(2)*X(3)
        VAL=-VAL
        GO TO 99
224     GO TO (31,32,33,34),IN
 31      VAL=42-X(1)
          GO TO 99
32      VAL=42-X(2)
         GO TO 99
33      VAL=42-X(3)
        GO TO 99
34      VAL=(X(1)+2*X(2)+2*X(3))-72
        VAL=-VAL
```

```
99      RETURN
        END
        SUBROUTINE PRINDS
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
        COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
        COMMON /DEVC/NI,NO,NP
        COMMON/BB/BOU(30)
        COMMON/BBC/NNP,MOPT,NOBJ
        COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
      2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
      3CM
        WRITE(9,595)
595     FORMAT(/,'FOR STRATEGY NO.1',/)
        WRITE(9,555) BOU(14)
555     FORMAT(/,'TOTAL DEPTH OF LAYER = ',F10.2,1X,'MM')
        WRITE(9,525) NNP
525     FORMAT(/,'NO OF PASSES = ',I2,/)
        WRITE(9,194) BOU(15)
194     FORMAT(/,'DEPTH OF CUT FOR EACH PASS = ',F10.2,1X,'MM')
        IF(NOBJ.EQ.2) GO TO 0302
        IF(NOBJ.EQ.3) GO TO 0305
        WRITE (9,008) F
008     FORMAT (/,2X,'TOTAL PRODUCTION COST = ',1PE16.8,1X,'YENS PIECE')
        GO TO 0303
0302    WRITE (9,108) F
108     FORMAT (/,2X,'TOTAL PRODUCTION TIME = ',1PE16.8,1X,'MINUTES')
        GO TO 0303
0305    WRITE (9,188) F
188     FORMAT (/,2X,'MATERIAL REMOVAL RATE = ',1PE16.8,1X,'M**3/MIN')
0303    WRITE (9,009)
009     FORMAT (/,2X,'FINAL X VALUES',/)
        WRITE (9,011) (I,X(I),I=1,N)
011     FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
        SF=(C6*X(1)**(A10)*X(2)**A11*BOU(15)**A12)
        WRITE (9,022)  SF
022     FORMAT (/,2X,'SURFACE FINISH = ',F10.2,1X,'MICROMETERS')
        TL=C1*X(1)**(B1)*2.718**(B3*X(2))*BOU(15)**B4
        WRITE (9,037)  TL
037     FORMAT (/,2X,'TOOL LIFE = ',F10.2,1X,'MINS')
        RETURN
        END
        SUBROUTINE PRISUM
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
        COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
        COMMON /DEVC/NI,NO,NP
```

```
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
      IF(NOBJ.EQ.2) GO TO 0302
      IF(NOBJ.EQ.3) GO TO 1
      WRITE(9,595)
595   FORMAT(/,25X,'FOR STRATEGY NO.5',/)
      WRITE(9,555) BOU(14)
555   FORMAT(/,'TOTAL DEPTH OF LAYER = ',F10.2,1X,'MM')
      WRITE(9,525) NNP
525   FORMAT(/,'NO OF DIVS OF DEPTH OF CUT = ',I2,/)
      WRITE (9,008) F
008   FORMAT (/,2X,'TOTAL PRODUCTION COST = ',1PE16.8,1X,'YENS PIECE')
      WRITE (9,009)
009   FORMAT (/,2X,'FINAL X VALUES',/)
      WRITE (9,011) (I,X(I),I=1,N)
011   FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
      KK=0
      DO 55 II=1,NNP
      I=II+KK
      J=I+1
      K=J+1
      SF=(C6*X(I)**(A10)*X(J)**A11*X(K)**A12)
      WRITE (9,022) II,SF
022   FORMAT (/,2X,'FOR PASS NO',I2,2X,'SURFACE FINISH = ',F10.2,1X,
     a'MICROMETERS')
      TL=(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)**B4)
      WRITE (9,052) II,TL
052   FORMAT (/,2X,'FOR PASS NO',I2,2X,'TOOL LIFE = ',F10.2,1X,'MINS')
      COST=ALPHAO*X(I)**(-1)*X(J)**(-1)+ALPHA1*X(I)**
     aBETA1*X(J)** BETA2*X(K)**BETA4*(2.718)**(BETA3*X(J))+CM
      WRITE (9,952) II,COST
952  FORMAT (/,2X,'FOR PASS NO',I2,2X,'COST = ',F10.2,1X,'YENS PIECE')
      KK=KK+2
55    CONTINUE
      RETURN
0302  WRITE(9,195)
195   FORMAT(/,25X,'FOR STRATEGY NO.5',/)
      WRITE(9,155) BOU(14)
155   FORMAT(/,'TOTAL DEPTH OF LAYER = ',F10.2,1X,'MM')
      WRITE(9,125) NNP
125   FORMAT(/,'NO OF DIVS OF DEPTH OF CUT = ',I2,/)
      WRITE (9,108) F
108   FORMAT (/,2X,'TOTAL PRODUCTION TIME = ',1PE16.8,1X,'MINUTES')
      WRITE (9,109)
```

```
109    FORMAT (/,2X,'FINAL X VALUES',/)
       WRITE (9,111) (I,X(I),I=1,N)
111    FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
       KK=0
       DO 551 II=1,NNP
       I=II+KK
       J=I+1
       K=J+1
       SF=(C6*X(I)**(A10)*X(J)**A11*X(K)**A12)
       WRITE (9,122) II,SF
122    FORMAT (/,2X,'FOR PASS NO',I2,2X,'SURFACE FINISH = ',F10.2,/)
       TL=(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)**B4)
       WRITE (9,152) II,TL
152    FORMAT (/,2X,'FOR PASS NO',I2,2X,'TOOL LIFE = ',F10.2,/)
       TIME=ALPHA0*X(I)**(-1)*X(J)**(-1)+ALPHA1*X(I)**
     aBETA1*X(J)** BETA2*X(K)**BETA4*(2.718)**(BETA3*X(J))+CM
       WRITE (9,912) II,TIME
912    FORMAT (/,2X,'FOR PASS NO',I2,2X,'TIME = '.F10.2,/)
       KK=KK+2
551    CONTINUE
       RETURN
1      WRITE(9,2)
2      FORMAT(/,25X,'FOR STRATEGY NO.5',/)
       WRITE(9,3) BOU(14)
3      FORMAT(/,'TOTAL DEPTH OF LAYER = ',F10.2,1X,'MM')
       WRITE(9,4) NNP
4      FORMAT(/,'NO OF DIVS OF DEPTH OF CUT = ',I2,/)
       WRITE (9,5) F
5      FORMAT (/,2X,'TOTAL MRR = ',1PE16.8,1X,'M**3/MIN')
       WRITE (9,6)
6      FORMAT (/,2X,'FINAL X VALUES',/)
       WRITE (9,7) (I,X(I),I=1,N)
7      FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
       KK=0
       DO 13 II=1,NNP
       I=II+KK
       J=I+1
       K=J+1
       SF=(C6*X(I)**(A10)*X(J)**A11*X(K)**A12)
       WRITE (9,19) II,SF
19     FORMAT (/,2X,'FOR PASS NO',I2,2X,'SURFACE FINISH = ',F10.2,/)
       TL=(C1*X(I)**(B1)*2.718**(B3*X(J))*X(K)**B4)
       WRITE (9,10) II,TL
10     FORMAT (/,2X,'FOR PASS NO',I2,2X,'TOOL LIFE = ',F10.2,/)
       SMRR=10E-6*X(I)*X(J)*X(K)
       WRITE (9,15) II,SMRR
15     FORMAT (/,2X,'FOR PASS NO',I2,2X,'SMRR = ',1PE16.8,'M**3/MIN')
```

```
      KK=KK+2
13   CONTINUE
      RETURN
      END
      SUBROUTINE PRIEXM
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON /DEVC/NI,NO,NP
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      WRITE (9,008) F
008   FORMAT (/,2X,'FINAL VALUE OF F = ',1PE16.8 )
      WRITE (9,009)
009   FORMAT (/,2X,'FINAL X VALUES',/)
      WRITE (9,011) (I,X(I),I=1,N)
011   FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
      RETURN
      END
      SUBROUTINE PRIAND
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
      COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
      COMMON /DEVC/NI,NO,NP
      COMMON/BB/BOU(30)
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/PARM/ALPHA0,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
    2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
    3CM
      IF(NNP) 277,277,278
278   WRITE(9,2689)
2689  FORMAT(/,'VALUES FOR BOTH ROUGHING AND FINISHING PASSES',/)
      WRITE(9,378)
378   FORMAT(/,'FOR STRATEGY NO.4',/)
      SF1=(C6*X(1)**(A10)*X(2)**A11*X(3)**A12)
      SF2=(C6*X(4)**(A10)*X(5)**A11*X(6)**A12)
     TL1=C1*X(1)**(B1)*2.718**(B3*X(2))*X(3)**B4
     TL2=C1*X(4)**(B1)*2.718**(B3*X(5))*X(6)**B4
     PR1=NNP*(ALPHA0*X(1)**(-1)*X(2)**(-1)+ALPHA1
    a*X(1)**BETA1*X(2)**
    a(BETA2)*X(3)**(BETA4)*(2.718)**(BETA3*X(2))+CM)
     PR2=ALPHA0*X(4)**(-1)*X(5)**(-1)+ALPHA1*X(4)**BETA1*X(5)**
    a(BETA2)*X(6)**(BETA4)*(2.718)**(BETA3*X(5))+CM
      WRITE(9,555) BOU(15)
555   FORMAT(/,'TOTAL DEPTH OF LAYER = ',F10.2,1X,'MM')
      WRITE(9,525) NNP
525   FORMAT(/,'NO OF ROUGHING PASSES = ',I2,/)
```

```
      IF(NOBJ.EQ.2) GO TO 0302
      IF(NOBJ.EQ.3) GO TO 0312
      WRITE (9,3) PR1
3     FORMAT (/,2X,'THE COST FOR ROUGHING = ',1PE16.8,1X,'YENS PIECE')
      WRITE (9,4) PR2
4     FORMAT (/,2X,'THE COST FOR FINISHING = ',1PE16.8,1X,'YENS PIECE')
      WRITE (9,008) F
008   FORMAT (/,2X,'THE TOTAL PRODUCTION COST = ',1PE16.8,1X,
     3'YENS PIECE')
      GO TO 0303
0302  WRITE (9,399) PR1
399   FORMAT (/,2X,'THE TIME FOR ROUGHING = ',1PE16.8,1X,'MINUTES')
      WRITE (9,499) PR2
499   FORMAT (/,2X,'THE TIME FOR FINISHING = ',1PE16.8,1X,'MINUTES')
      WRITE (9,908) F
908   FORMAT (/,2X,'THE TOTAL PRODUCTION TIME = ',1PE16.8,1X,
     3'MINUTES')
      GO TO 0303
0312  PR1=10E-6*X(1)*X(2)*X(3)
      WRITE (9,490) PR1
490   FORMAT (/,2X,'MRR FOR ROUGHING = ',1PE16.8,1X,'M**3/MIN')
      PR2=10E-6*X(4)*X(5)*X(6)
      WRITE (9,599) PR2
599   FORMAT (/,2X,'MRR FOR FINISHING = ',1PE16.8,1X,'M**3/MIN')
      WRITE (9,958) F
958   FORMAT (/,2X,'TOTAL MRR = ',1PE16.8,1X,'M**3/MIN')
0303  WRITE (9,009)
009   FORMAT (/,2X,'FINAL X VALUES',/)
      WRITE (9,011) (I,X(I),I=1,N)
011   FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
      WRITE(9,77) SF1
77    FORMAT(/,'SURFACE FINISH FOR ROUGHING PASSES = ',F10.2,1X,
     a'MICROMETERS')
      WRITE(9,88) SF2
88    FORMAT(/,'SURFACE FINISH FOR THE FINISHING PASS = ',F10.2,1X,
     a'MICROMETERS')
      WRITE(9,1) TL1
1     FORMAT(/,'TOOL LIFE FOR THE ROUGHING PASSES = ',F10.2,1X,'MINS')
      WRITE(9,2) TL2
2     FORMAT(/,'TOOL LIFE FOR FOR THE FINISHING PASS = ',F10.2,1X,
     a'MINS')
      RETURN
277   WRITE(9,2687)
2687  FORMAT(/,'VALUES FOR FINISHING PASS ONLY',/)
      WRITE(9,5550) BOU(15)
5550  FORMAT(/,'TOTAL DEPTH OF LAYER = ',F10.2,1X,'MM')
      WRITE(9,5250) NNP
```

```
5250   FORMAT(/,'NO OF ROUGHING PASSES = ',I2,/)
       WRITE (9,808) F
808    FORMAT (/,2X,'THE TOTAL PRODUCTION COST = ',1PE16.8,1X,
     3'YENS PIECE')
       WRITE (9,909)
909    FORMAT (/,2X,'FINAL X VALUES',/)
       WRITE (9,111) (I,X(I),I=1,N)
111    FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
       SF=(C6*X(1)**(A10)*X(2)**A11*BOU(15)**A12)
       WRITE(9,66) SF
66     FORMAT(/,'SURFACE FINISH = ',F8.2)
       RETURN
       END
       SUBROUTINE MAIN2
       IMPLICIT REAL*8(A-H,O-Z)
       COMMON /SHARE/ X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
       COMMON /EQAL/H,H1,MZ
       COMMON/OPTNS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
       COMMON/VALUE/F,G,PO,RSIGMA,RJ(200),RHO
       COMMON/CRST/DELX(100),DELXO(100),RHOIN,RATIO,EPSI,THETAO,
     1RSIG1,G1,X1(100),X2(100),X3(100),XR2(100),XR1(100),PR1,
     2PR2,P1,F1,RJ1(200),DOTT,PGRAD(100),DIAG(100),
     3PREV3,ADELX,NTCTR,NUMINI,NPHASE,NSATIS
       COMMON /EXPOPT/ NEXOP1,NEXOP2,XEP1,XEP2
       COMMON /DEVC/NI,NO,NP
       COMMON/BB/BOU(30)
       COMMON/BBC/NNP,MOPT,NOBJ
       COMMON/PARM/ALPHAO,ALPHA1,BETA1,BETA2,BETA3,BETA4,C1,C2,
     2C3,C4,C5,C6,A1,A2,A3,A5,A6,A7,A8,A9,A10,A11,A12,B1,B2,B3,B4,
     3CM
       DIMENSION FF(20),XOPT(600)
       READ(NI,*) MNNP
       NNP=0
       GO TO 0104
0105   NNP=NNP+1
       IF (NNP.EQ.MNNP) GO TO 0607
0104   READ(NI,*)EPSI,RHOIN,THETAO,RATIO,TMMAX
       READ(NI,*) M,N,MZ
       IF (N) 40,40,107
107    CONTINUE
       READ(NI,*)NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
       READ(NI,*) XEP1,XEP2
       READ (NI,*) NEXOP1,NEXOP2
       READ (NI,*) MJJ
       DO 2834 MNJ=1,MJJ
       READ (NI,*) BOU(MNJ)
2834   CONTINUE
```

```
      READ(NI,*) (X(I),I=1,N)
      WRITE(NO,004)
004   FORMAT (1H1,10X,'NONLINEAR PROGRAMMING ROUTINE  -- . SUMT
     1VERSION',/)
      WRITE(NO,005) N,M,MZ,TMMAX,RHOIN,RATIO,EPSI,THETAO
005   FORMAT (//,2X,10HPARAMETERS,//,2X,4HN = ,I2,4X,4HM = ,I2,4X,5HMZ
     1 = ,I2,4X,8HTMMAX = ,F8.3,/,2X,6HRHO = ,E10.4,4X,8HRATIO = ,E10.4,
     24X,9HEPSILON = ,E11.4,4X,8HTHETA = ,E10.4 )
      WRITE (NO,006) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10,NEXOP1,
     1NEXOP2
006   FORMAT (//,2X,16HOPTIONS SELECTED,//,2X,6HNT1 = ,I1,3X,6HNT2 = ,
     1I1,3X,6HNT3 = ,I1,3X,6HNT4 = ,I1,3X,6HNT5 = ,I1,/,2X,6HNT6 = ,
     2I1,3X,6HNT7 = ,I1,3X,6HNT8 = ,I1,3X,6HNT9 = ,I1,3X,7HNT10 = ,I1,
     3/,2X,9HNEXOP1 = ,I1,3X,9HNEXOP2 = ,I1 )
      WRITE (NO,007) XEP1,XEP2
007   FORMAT (//,2X,37HTOLERANCES FOR DIFFERENCING AND MOVES ,//,2X,
     17HXEP1 = ,E10.4,4X,7HXEP2 = ,E10.4 )
      NTCTR=0
      NP1=N+1
      NM1=N-1
      CALL TIMEC
      NPHASE=4
      CALL EVALU
      PO=0.0
      G=0.0
      H =0.0
      RSIGMA=0.0
      CALL OUTPUT (2)
      CALL STORE
      IF (NEXOP1.GT.1) CALL CHCKER
      IF (NEXOP1.EQ.3) GO TO 40
      IF (NEXOP1.EQ.5) GO TO 40
      CALL FEAS
      WRITE(7,0908) NPHASE
0908  FORMAT(/,I2,/)
      GO TO (30,30,30,30,0501) ,NPHASE
30    NPHASE=2
      NTCTR=0
      CALL BODY
      IF(NNP) 277,277,278
277   WRITE(NO,2687)
2687  FORMAT(/,'VALUES FOR FINISHING PASS ONLY',/)
      GO TO 279
278   WRITE(NO,2689)
2689  FORMAT(/,'VALUES FOR BOTH ROUGHING AND FINISHING PASSES',/)
279   WRITE(NO,555) BOU(15)
555   FORMAT(/,'TOTAL DEPTH OF CUT = ',F10.2)
```

```
      WRITE(NO,525) NNP
525   FORMAT(/,'NO OF ROUGHING PASSES = ',I2,/)
      WRITE (NO,008) F
008   FORMAT (/,2X,'FINAL VALUE OF F = ',1PE16.8 )
      WRITE (NO,009)
009   FORMAT (/,2X,'FINAL X VALUES',/)
      WRITE (NO,011) (I,X(I),I=1,N)
011   FORMAT (/,3(2X,2HX(,I2,4H) = ,1PE16.8 ),//)
C.FOR FINDING OUT THE MINIMUM VALUE OF OBJECTIVE FUNCTION
      IF(N.EQ.2) GO TO 0116
      IF(N.GT.2) GO TO 0117
0116  DO 0118 I=3,6
      XOPT(I)=0.0
0118  CONTINUE
0117  DO 0114 I=1,N
      IJ=I+6*NNP
      XOPT(IJ)=X(I)
0114  CONTINUE
      II=NNP+1
      FF(II)=F
      GO TO 0502
0501  KJ=NNP+1
      KK=KJ-1
      IF (NNP) 0601,0601,0602
0601  FF(KJ)=1E5
      DO 0603 I=1,6
      XOPT(I)=0.0
0603  CONTINUE
      GO TO 0105
0602  FF(KJ)=FF(KK)
      DO 0604 I=1,N
      MN=I+6*NNP
      MM=I+6*NNP-6
      XOPT(MN)=XOPT(MM)
0604  CONTINUE
      IF(NNP.LT.MNNP) GO TO 0105
0502  CONTINUE
      IF(NNP.EQ.0) GO TO 0105
      IF(NNP.GE.1) CONTINUE
      JJ=II-1
      IF(FF(II).LE.FF(JJ)) GO TO 0105
      IF(FF(II).GT.FF(JJ)) CONTINUE
      WRITE(9,2688)
2688  FORMAT(10X,'**************************',/)
      WRITE(9,0113) BOU(15)
0113  FORMAT(/,'TOTAL DEPTH OF CUT = ',F10.2)
      NNP1=NNP-1
```

```
      WRITE(9,0111) NNP1
0111  FORMAT(/,'THE OPTIMUM NO OF  ROUGHING PASSES = ',I2,/)
      WRITE(9,0112) FF(JJ)
0112  FORMAT(/,'THE MINIMUM PRODUCTION COST = ',F15.5,/)
      IF (NNP1) 0204,0204,0208
0204  N=2
      WRITE(9,0205)
0205  FORMAT (/,'THE OPTIMUM VALUES OF SPEED AND FEED FOR FINISHING
     @PASS ONLY',/)
      GO TO 0206
0208  WRITE(9,0201)
0201  FORMAT (/,'THE OPTIMUM VALUES OF SPEED,FEED AND DEPTH OF CUT FOR
     @ROUGHING AND FINISHING PASSES RESPECTIVELY',/)
0206  DO 0115 I=1,N
      JI=I+6*NNP1
      WRITE(9,0202) XOPT(JI)
0115  CONTINUE
0202  FORMAT(/,1PE16.8,/)
      WRITE(9,0203)
0203  FORMAT(10X,'*************************',/)
      GO TO 40
0607  WRITE(9,0702)
0702  FORMAT(/,'EITHER THERE IS NO FEASIBLE POINT OR A FEW OF THE
     @POINTS ARE FEASIBE',/)
      WRITE(9,0712) FF(NNP)
0712  FORMAT(/,'THE MINIMUM PRODUCTION COST = ',F15.5,/)
      WRITE(9,0801)
0801  FORMAT (/,'THE OPTIMUM VALUES OF SPEED,FEED AND DEPTH OF CUT FOR
     @ROUGHING AND FINISHING PASSES RESPECTIVELY',/)
      DO 0715 I=1,N
      JI=I+6*(NNP-1)
      WRITE(9,0202) XOPT(JI)
0715  CONTINUE
40    RETURN
      END
      SUBROUTINE MODF
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /SHARE/ X(100),DEL(100),A(100,100).N,M,MN,NP1,NM1
      COMMON/BBC/NNP,MOPT,NOBJ
      COMMON/EXTM/M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14,M15
      M1=(M-2)/NNP
      M2=2*M1
      M3=3*M1
      M4=4*M1
      M5=5*M1
      M6=6*M1
      M7=7*M1
```

```
M8=8*M1
M9=9*M1
M10=10*M1
M11=11*M1
M12=12*M1
M13=13*M1
M14=14*M1
M15=15*M1
RETURN
END
```

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

1) An extensive review of the machining optimization models has been carried out. Optimization methods pertinent to machining optimization have been compiled and evaluated by applying them to a number of machining models selected from the compiled machining optimization models. All the optimization methods considered, except GOMTRY, can handle the machining optimization problem. Sequential Unconstrained Minimization Technique (SUMT) and personal computer version of the Generalized Reduced Gradient (GRG2), the Generalized Integrated Optimizer (GINO), were found to be most suitable for application in machining economics optimization.

2) For a given machining optimization model, the results of optimization depend upon the machining strategy employed. Five strategies for multipass machining optimization have been proposed. The results obtained for the proposed multipass machining strategies for cost minimization of multipass turning operation, using a general model, are found to be sensitive to the total depth of material to be removed, the surface finish constraint in the last pass and the tool life constraints in the roughing and the finishing passes.

3) Of the five proposed machining strategies for multipass turning, MS5 was found to be the most economical and general in nature. However, computational savings with MS4 combined with the conjectured equivalence of MS5 to MS4 dictates the preference of MS4 over MS5.

4) A general computer program package based on SUMT optimization method, which runs on both the mainframe and personal computer, has been developed to handle generalized multipass machining optimization problems. The package allows the user the choice of objective function, choice of constraints and the choice of a cutting strategy. With some modifications, the developed program package is also capable of handling multi-tool and multi-operation optimization problems.

6.2 RECOMMENDATIONS

1) The equality of results for MS4 and MS5 strategies for multipass turning in chapter 4 lead to the conjecture that MS4 is equivalent to MS5. It is recommended to pursue an analytical proof of the equivalence of MS4 to MS5.

2) For MS2, the production cost increases with increasing 'A' (total depth of material to be removed) up to a certain value of 'A', where the production cost drops and then increases again with 'A'. This phenomenon which could be related to 'more for less paradox' warrants further investigation.

3) The model considered for optimization in this thesis is deterministic in nature. The results of the proposed machining strategies can be extended to the probabilistic models in a future work.

4) Modification of the developed package for its interactive use on the personal computer for optimization of multipass, multi-operation or multi-tool machining models is recommended to make it suitable for industrial application.

5) The developed computer optimization program package is proposed for use with NC programming applications for selecting optimal cutting conditions for NC programs by making it complementary to the NC processor packages such as APT (Automatically Programmed Tools).

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APPENDIX A

This appendix presents a selected number of constraint formulations from [25,26,39,40,41,42].

A) Constraint Formulations for Cutting Force :

The following have been taken from [25].

a) The Cutting Force relations derived from Taylor's Cutting Force tests for different materials are given as:

Hard Cast Iron:

$$F_c = 244\left(\frac{G}{5}\right)^{0.092} (1000A)^{0.841} \text{ lbs} \tag{A.1}$$

Soft Cast Iron:

$$F_c = 156\left(\frac{G}{5}\right)^{0.092} (1000A)^{0.841} \text{ lbs} \tag{A.2}$$

Medium Steel:

$$F_c = 305\left(\frac{G}{5}\right)^{0.034} (1000A)^{0.966} \text{ lbs} \tag{A.3}$$

b) A number of tests were made on SAE 1020 to determine the effect of changes of tool geometry (true rake and side cutting edge angle) on the Cutting Force.

i) The expressions for the Cutting Force, for SAE 1020 with changes in true rake angle (degrees) are:

Back rake angle 8, side rake 6, side cutting edge angle 0, Tool nose radius 3/64 inches and true rake angle=6:

$$F_c = 300(\frac{G}{5})^{0.092} (1000A)^{0.908} \text{ lbs} \tag{A.4}$$

Back rake angle 8, side rake 14, side cutting edge angle 0, Tool nose radius 3/64 inches and True rake angle=14:

$$F_c = 270(\frac{G}{5})^{0.088} (1000A)^{0.912} \text{ lbs} \tag{A.5}$$

Back rake angle 8, side rake 22, side cutting edge angle 0, Tool nose radius 3/64 inches and True rake angle=22:

$$F_c = 240(\frac{G}{5})^{0.01} (1000A)^{0.9} \text{ lbs} \tag{A.6}$$

Where 'G' is the slenderness ratio i.e.Depth of Cut/feed, 'A' is the chip X-Sectional area in thousandths of an inch.

ii) The effect of side cutting edge angle on the Cutting Force:

Back rake 8, side rake 14, true rake 16 1/4, nose radius 3/64 inches and S.C.E.A.=30 :

$$F_c = 282(\frac{G}{5})^{0.100} (1000A)^{0.900} \text{ lbs} \tag{A.7}$$

Back rake 8, side rake 14, true rake 15 1/2, nose radius 3/64 inches and S.C.E.A.=45 :

$$F_c = 314(\frac{G}{5})^{0.095} (1000A)^{0.835} \text{ lbs} \tag{A.8}$$

iii) The expressions for Cutting Force for different tool nose radii are:

For Back rake 8, side rake 14, True rake 14 and side cutting edge angle 0.

Tool nose radius= 1/32 inch :

$$F_c = 267(\frac{G}{5})^{0.008} (1000A)^{0.920} \text{ lbs} \tag{A.9}$$

Tool nose radius= 3/16 inch :

$$F_c = 366(\frac{G}{5})^{0.075} (1000A)^{0.755} \text{ lbs} \tag{A.10}$$

Tool nose radius= 1/4 inch :

$$F_c = 530(\frac{G}{5})^{0.13} (1000A)^{0.66} \text{ lbs} \tag{A.11}$$

c) Making use of the data derived from tests at the Berlin Technical

University, Cutting Force is expressed as a combined function of Chip X-section, Brinell Hardness and Tool Geometry:

Cr Ni Steel :

$$F_c = 40\ (1000A)^{0.802}\ (BHN)^{0.456}\ \left(\frac{\beta}{50}\right)^{0.64}\ \text{lbs} \qquad (A.12)$$

where β is the lip angle of the tool.

SAE 1060 :

$$F_c = 40\ (1000A)^{0.803}\ (BHN)^{0.454}\ \left(\frac{\beta}{50}\right)^{0.595}\ \text{lbs} \qquad (A.13)$$

Wrought Iron :

$$F_c = 29.6\ (1000A)^{0.862}\ (BHN)^{0.457}\ \left(\frac{\beta}{50}\right)^{0.735}\ \text{lbs} \qquad (A.14)$$

Cast Iron :

$$F_c = 14.5\ (1000A)^{0.865}\ (BHN)^{0.398}\ \left(\frac{\beta}{50}\right)^{0.663}\ \text{lbs} \qquad (A.15)$$

d) For machining steel similar to SAE 1020, the Extended Cutting Force law computed from Cave's Tests at the 'Laboratoire Central d'Armements' in France is given by:

Feed Force:

$$P_1 = 182\ (1\text{-}0.017\alpha)\ (1000A)^{0.945}(\frac{G}{5})^{0.205}\ \text{lbs} \qquad (A.16)$$

where α is the true rake angle of the tool.

Radial Force :

$$P_2 = 228\ (1\text{-}0.018\alpha)\ (1000A)^{0.760}(\frac{G}{5})^{0.140}\ \text{lbs} \qquad (A.17)$$

Main Cutting Force :

$$P_3 = 444\ (1\text{-}0.013\alpha)\ (1000A)^{0.770}(\frac{G}{5})^{0.150}\ \text{lbs} \qquad (A.18)$$

Note: Although the feed force and the radial force have no substantial effect on the horsepower requirements and the metal removal rate, these minor cutting force components play an important role in the deformation, the accuracy and vibration of machine tools.

The following have been taken from [26].

a) Tool Point Material is High Speed Steel:

i) For the workpiece material of carbon steel, with a tensile strength of 75 kg/ mm^2, Brinell Hardness of 215 kg/ mm^2, the Tangential, Radial and Feed components of Cutting Force respectively in Kilograms are expressed as:

$$F_v = 200as^{0.75} \tag{A.19}$$

$$F_p = 124.8a^{0.9}s^{0.75} \tag{A.20}$$

$$F_f = 66.8a^{1.2}s^{0.65} \tag{A.21}$$

'a', the depth of cut, ranges from 1 to 1.2 mm and 's', the feed, ranges from 0.1 to 2.0 mm.

ii) For the workpiece material of Gray cast Iron, with a Brinell Hardness of 190 kg/ mm^2, the three components of Force are:

$$F_v = 114as^{0.75} \tag{A.22}$$

$$F_p = 119.2a^{0.5}s^{0.75} \tag{A.23}$$

$$F_f = 51.4a^{1.2}s^{0.65} \tag{A.24}$$

'a' ranges from 1 to 1.5 mm and 's' ranges from 0.1 to 3.0 mm.

iii) For the workpiece material of Nodular cast Iron, with a Brinell

Hardness of 190 kg/ mm^2, the Tangential component of force is expressed as:

$$F_v = 153.0a^{1.03}s^{0.94} \qquad (A.25)$$

'a' ranges from 1 to 8.0 mm and 's' ranges from 0.1 to 2.0 mm.

iv) For the workpiece material of Malleable cast Iron, with a Brinell Hardness of 150 kg/ mm^2, the three components of force are:

$$F_v = 100as^{0.75} \qquad (A.26)$$

$$F_p = 87.60a^{0.9}s^{0.75} \qquad (A.27)$$

$$F_f = 39.6a^{1.2}s^{0.65} \qquad (A.28)$$

'a' ranges from 1 to 8.0 mm and 's' ranges from 0.1 to 1.4 mm.

b) Tool Point Material is Sintered Carbide:

i) For the workpiece material of carbon steel,with a tensile strength of 75 kg/ mm^2, Brinell Hardness of 215 kg/ mm^2, the Tangential component of force is expressed as:

$$F_v = 191.0as^{0.75} \qquad (A.29)$$

'a' ranges from 1 to 1.2 mm and 's' ranges from 0.1 to 2.0 mm.

ii) For the workpiece material of Hardened steel, with a tensile strength of 150 kg/ mm^2, the Tangential component of force is expressed as:

$$F_v = 300.0a^{0.81}s^{0.81} \tag{A.30}$$

'a' ranges from 1 to 2.0 mm and 's' ranges from 0.1 to 1.5 mm.

iii) For the workpiece material of Gray Cast Iron with a Brinell Hardness of 190 kg/ mm^2, the Tangential component of force is expressed as:

$$F_v = 92as^{0.75} \tag{A.31}$$

'a' ranges from 1 to 1.5 mm and 's' ranges from 0.1 to 3.0 mm.

iv) For the workpiece material of Nodular Cast Iron with a Brinell Hardness of 190 kg/ mm^2, the Tangential component of force is expressed as:

$$F_v = 145a^{0.92}s^{0.82} \tag{A.32}$$

'a' ranges from 1 to 8.0 mm and 's' ranges from 0.1 to 2.0 mm.

The following Empirical formula for Cutting Force in Turning has been taken from [41]:

$$F_c = C_f \, f^a \, c^b \tag{A.33}$$

Average value found for 'a' is around 0.8 and for 'b' it is about 0.9. Some average values for C_f for cutting force in kN and 'f' and 'C' in mm are as follows:

MATERIAL	BHN	C_f
Low C Steel	100	2.18
M alloy steel	207	4.00
Cast Iron	126	0.53

B) Constraint Formulations for Surface Finish :

The following has been taken from [42].

$$R_a = 1.36E8s^{1.004}V^{-1.52} \tag{A.34}$$

R_a is the CLA value in μinches, 's' is the feed in inch per rev, and 'V' is the speed in fpm, for $75 \le V \le 750$ and $s < 0.03$.

$$R_a = 7.34E4s^{1.54} \tag{A.35}$$

for $s < 0.03$ and $V > 750$.

$$R_a = 5.1E9s^{4.54} \tag{A.36}$$

for all cutting speeds and $s > 0.03$.

The Equations (A.34) to (A.36) are valid when tool nose radius is 0.04 inches and Brinell Hardness of the workpiece material is 195.

Taking into account the workpiece Brinell Hardness H_B and φ, the principal cutting edge angle, the generalized expressions for surface roughness are:

$$R_a = 11.78E9s^{4.54}r^{-0.714}H_B^{-0.323}\varphi^{-0.35} \tag{A.37}$$

for s > 0.03

$$R_a = 3.14E8s^{1.004}V^{-1.52}r^{-0.714}H_B^{-0.323}\varphi^{-0.35} \quad (A.38)$$

for 75 ≤ V ≤ 750 and s < 0.03

$$R_a = 16.98E4s^{1.54}r^{-0.714}H_B^{-0.323}\varphi^{-0.35} \quad (A.39)$$

for s < 0.03 and V > 750

The following has been taken from [39].

$$R_a = \frac{f}{4(\cot K_{r_e} + \cot K_{r_e}')} \ \mu\text{meters} \quad (A.40)$$

where f is the feed in mm, K_{r_e} is the major cutting edge angle , and K_{r_e}' is the minor cutting edge angle.

The following has been taken from [40].

$$R_{th}' = \frac{t^2}{8r} + \frac{t_m}{2}(1 + \frac{rt_m}{2}) \quad (A.41)$$

where R_{th}' is the peak to valley height, 't' is the feed in turning in mm, t_m is the Maximum Undeformed chip thickness in mm and 'r' is the nose radius of the tool in mm.